

Math 180 Midterm 2 Review

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These are probably a bit harder than what will show up on the test, but if you understand how to work these problems then you should understand everything on the test.

1. Sketch the graph of the *Witch of Agnesi*

$$y = \frac{8}{x^2 + 4}.$$

Indicate local minima/maxima and inflection points, and make sure all sections of the curve have the proper concavity.

2. Write the equations for the tangent lines to the *three-petaled rose*

$$(x^2 + y^2)^2 = x^3 - 3xy^2$$

at the points $\left(\frac{-1}{2}, \frac{\pm\sqrt{3}}{2}\right)$. Can you figure out what the tangent line is at the point $(0, 1)$? If you conclude that it doesn't exist, you're wrong. (Possible hint for the last part: try calculating dx/dy and think about what this means. Or you could cheat and graph it with a computer.)

3. Derive the formula for $\arctan x$, given that the derivative of $\tan x$ is $\sec^2 x$. (The point is to complete the derivation correctly, not just to get the right answer.)
4. Approximate $0.972^{0.972}$ by taking a linear approximation of the function $f(x) = x^x$ at $x = 1$.
5. If $g(x) = \sin(2^x) \ln(x)$, what are $f'(x)$ and $f''(x)$?
6. A water balloon remains perfectly spherical while water is added to it at a constant rate of $5\text{cm}^3/\text{s}$. If the balloon is currently 12 cm across, how rapidly is its diameter currently increasing?
7. The customers at your pizza shop are willing to pay \$8 for an 8" diameter pizza, \$14 for a 14" diameter pizza, and so forth. Pizza toppings cost you five cents per square inch. What is your favorite size of pizza to sell? (Assume you are motivated exclusively by greed.)
8. Use the Mean Value Theorem to prove that $|\cos(a) - \cos(b)| \leq |a - b|$ for all real numbers a and b . (Hint: Apply the Mean Value Theorem to $f(x) = \cos(x)$ on the interval $[a, b]$ and think about the meaning of the resulting statement.)
9. Sketch a graph of the function $y = \arctan(x)$. Indicate any local extrema, inflection points, and asymptotes. If you claim something is an asymptote, make sure to prove that it is actually an asymptote. (You will likely be expected to do all this any time you're told to sketch a graph, even if you're not explicitly asked to. If you're ever not 100% sure how much you need to do on any particular problem on the midterm or final, *please* raise your hand and ask one of us!)
10. Find the point on the line $y = 3x + 1$ closest to $(5, 1)$.