Note to class

Daniel McLaury

September 23, 2012

I got a question in section this week which came down to finding $\frac{d}{dx}e^{4x}$ (or something of the sort). I said that we'd have to go over the chain rule in order to work this problem, and promised to spend some time talking about it in class on Tuesday. Now, this is really how someone with a bit of experience with calculus would work this problem – if you know the chain rule, it's straightforward to calculate that

$$\frac{d}{dx}e^{4x} = e^{4x}\frac{d}{dx}4x = 4e^{4x}.$$

The chain rule is a very nice, general technique that lets you solve a whole lot of different kinds of problems quickly and easily.

However, it turns out that it's possible to find this particular derivative even if you're not familiar with the chain rule, and this was apparently the intended solution in this case. In particular, we can just use the product rule together with basic properties of the exponential function. Let's start out by calculating the derivative of e^{2x} and working our way up.

First, notice that $e^{2x} = e^x \cdot e^x$, so we can calculate the derivative using the product rule:

$$\frac{d}{dx}e^{2x} = \frac{d}{dx}(e^x \cdot e^x) = e^x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(e^x) = 2e^{2x}.$$

In turn, we have

$$\frac{d}{dx}e^{3x} = \frac{d}{dx}(e^x \cdot e^{2x}) = e^x\frac{d}{dx}(e^{2x}) + e^{2x}\frac{d}{dx}(e^x) = 2e^{3x} + e^{3x} = 3e^{3x}.$$

And of course you can imagine what the next step looks like.