Discussion Problems for Math 180

Tuesday, December 2, 2014

1. Sketch a graph of the function
$$f(x) = \frac{\sqrt[3]{x^3+1}}{x}.$$
Label any critical points, inflection points, horizontal or vertical asymptotes, etc. Provide complete justification for everything.

2. Recall that, for a real number x, floor(x) is the greatest integer less than or equal to x, and ceiling(x) is the smallest integer greater than or equal to x. Sketch a graph of the function
$$g(x) = \text{ceiling}(x) - \text{floor}(x)$$
on the domain $[-5, 5]$.

3. Calculate
$$\lim_{x\to 1} \frac{\frac{1}{x} - 1}{x - 1}.$$

4. Interpret your answer to number three as the derivative of a function at a point. Then calculate this derivative using derivative rules and check that your answer agrees.

5. Consider the function
$$h(x) = \begin{cases} 1 & \text{if } x < 0, \\ ax + 1 & \text{if } 0 \le x < 1, \\ x^2 - x + a + 1 & \text{if } 1 \le x. \end{cases}$$
For which values of a, if any, is this function continuous? Differentiable?

6. Estimate $\sqrt{62}$ using a local linear approximation.

7. Find all points on the curve
$$y^2 = 2x^2 - x^4 + 8$$
where the tangent line is either horizontal or vertical. *(Hint: there are eight such points in all.)*

8. Calculate derivatives:
   (a) $\sin^2(x)\cos^2(x)$
   (b) $\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$
   (c) $3\arctan(4x) - 4\arctan(3x)$
   (d) $\frac{\sqrt{x^2-1}}{x}$
   (e) $\left(\frac{x^4-1}{x^4+1}\right)^8$