Discussion Problems for Math 180

Thursday, December 4, 2014

1. Sketch the graph of a continuous function $h(t)$ such that:

   - the domain of h is $(0, \infty)$;
   - $\lim\limits_{t \to \infty} h(t) = 2$;
   - $h(2) = 3$; and
   - $h'(2) = -1$.

2. (a) Write down the definition of the derivative of a function $f(x)$ at a point a.

   (b) Calculate the derivative of $f(x) = \dfrac{1}{x}$ at $x = 5$ directly from the definition.

3. (a) Write down the definition of the integral of a function $f(x)$ over an integral $[a, b]$.

   (b) Calculate the integral of $f(x) = 2x - 1$ over the interval $[5, 6]$ directly from the definition.

4. (a) State the fundamental theorem of calculus.

   (b) Find $g'(x)$, where

$$g(x) := \int_{-x}^{x} \sin^8(t)\, dt.$$

5. (a) How many cubic centimeters in a liter?

   (b) A spherical balloon is inflated at a rate of one liter per hour. When the balloon is ten centimeters across, how quickly is its surface area increasing?

6. Write the equation of the tangent line to the function $f(x) = -2\sin^2(x)$ at the point where $x = \pi/6$.

7. Calculate integrals:

   (a) $\displaystyle \int \sin^2(t)\cos(t)\, dt$

   (b) $\displaystyle \int \frac{1}{\sqrt{1 - x}}\, dx$

   (c) $\displaystyle \int 2^x\, dx$

   (d) $\displaystyle \int \sec^2(x)\, dx$

   (e) $\displaystyle \int (x + 14)^8\, dx$

   (f) $\displaystyle \int \frac{2}{x^2 - 100}\, dx$

   (g) $\displaystyle \int \frac{2}{x^2 + 100}\, dx$