Solutions to Discussion Problems for Math 180

Tuesday, September 30, 3014

Remember to include units in answers where appropriate!

1. Have you reviewed trigonometry?

   - Express $\sin(\alpha+\beta)$ in terms of the sine and cosine of α and β.
     $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.
   - What are the sine and cosine of 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$? (Make a table.)

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

   - What is $\sin\left(\frac{5\pi}{12}\right)$?

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{6} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

2. Suppose you want to start a business selling muffins. The total cost, in dollars, to produce n muffins is given by $C = \$7,224 + \$0.05n$.

   (a) What is the average cost per muffin if you produce a thousand muffins?

$$\frac{\$7,224 + \$0.05(1,000)}{1,000} = \frac{\$7,274}{1,000} \approx \$7.27$$

   (b) ... if you produce a million?

$$\frac{\$7,224 + \$0.05(1,000,000)}{1,000,000} = \frac{\$57,224}{1,000,000} \approx \$0.06$$

   (c) What is the marginal cost of the thousandth muffin?

$$C'(n) = \$0.05 \Rightarrow C'(1,000) = \$0.05$$

   (d) ... the millionth?

$$C'(1,000,000) = \$0.05$$

(e) If people are willing to pay \$3.49 per muffin, how many would you have to sell to break even?

$$\$7,224 + \$0.05n = \$3.49n$$
$$n = \frac{\$7,224}{\$3.44} = 2100$$

3. What is the derivative of $f(x) = \sin^8(x)$?

$$f'(x) = 8\sin^7(x)\cos(x)$$

4. What is the derivative of $g(x) = \sqrt{1+x^2}$?

$$g'(x) = \frac{1}{2}(1+x^2)^{-1/2}(2x) = \frac{x}{\sqrt{1+x^2}}$$

5. Find the derivative of $h(x) = (1+x)^4$ in two different ways – by multiplying it out first and by using the chain rule – and demonstrate that you get the same answer both ways.

Expanding, we have

$$h(x) = (1+x)^2 = 1 + 4x + 6x^2 + 4x^3 + x^4 \Rightarrow h'(x) = 4x^3 + 12x^2 + 12x + 4$$

Or, using the chain rule,

$$h'(x) = 4(1+x)^3 = 4(1 + 3x + 3x^2 + x^3) = 4x^3 + 12x^2 + 12x + 4$$

6. On the rare occasions I have to deal with particularly troublesome students, I scale the outside of University Hall (which is 336 feet tall) with a backpack full of water balloons and wait for the trouble-maker to pass underneath. From basic physics, we know that after t seconds the height, in feet, of an object dropped from that height is given by $h(t) = 336 - 16t^2$.

(a) When releasing a balloon, I need to compensate for the time it takes to get to the ground. How long would that be, exactly?

We want

$$h(t) = 336 - 16t^2 = 0 \Rightarrow t^2 = 336/16 = 21$$

So it would take $\sqrt{21}$ seconds, which is between 4 and 5 seconds.

(b) How fast will the balloon be moving when it hits the ground?

$$h'(t) = -32t \Rightarrow h'(\sqrt{21}) = -32(\sqrt{21})$$

So it will be going at $32\sqrt{21} \approx 147$ ft/s. (On a test you'd just write $32\sqrt{21}$ ft/s – I don't expect you to take a square root by hand.)