Solutions for Discussion Problems

Wednesday, October 21

1. (a)
$$\frac{d}{dx}\left(\frac{\log_{10} x}{10^x}\right) = \frac{10^x\left(\frac{1}{x\ln(10)}\right) - (\log_{10} x)(10^x \ln(10))}{(10^x)^2}$$
   (b)
$$\frac{d}{dx}\cos^{-1}(5x) = \frac{-1}{\sqrt{1-(5x)^2}}\cdot 5$$
   (c)
$$\begin{aligned}\frac{d}{dx}\cos(x^{\cos(x)}) &= -\sin(x^{\cos(x)})\frac{d}{dx}(x^{\cos(x)})\\ &= -\sin(x^{\cos(x)})\frac{d}{dx}(e^{\cos(x)\ln(x)})\\ &= -\sin(x^{\cos(x)})e^{\cos(x)\ln(x)}\frac{d}{dx}(\cos(x)\ln(x))\\ &= -\sin(x^{\cos(x)})x^{\cos(x)}\left(-\sin(x)\ln(x)+\cos(x)\frac{1}{x}\right)\end{aligned}$$

2. Since $xy = 15$, $y = 15/x$, so we're minimizing
$$3x + 5y = 3x + 5(15/x) = 3x + 75/x$$
on $(0, \infty)$. Write $f(x) = 3x + 75/x$. We have
$$f'(x) = 3 - \frac{75}{x^2}$$
so there's a critical point at $x = 5$. (Also at $x = -5$ but that's outside our interval.) Now
$$f(5) = 3 \cdot 5 + 75/5 = 30.$$
Checking our endpoints,
$$\lim_{x\to 0^+} f(x) = \lim_{x\to\infty} f(x) = +\infty$$
So the minimum value is 30 at $x = 5$ and $y = 3$ and there's no maximum.

3. (a) $x = -3$ and $x = -1$
   (b) Increasing on $(1, \infty)$ and decreasing elsewhere.
   (c) Local min at $x = -1$.

4. (a)

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\begin{aligned}
g(x) &= 9x^{1/3} + 4 \\
g'(x) &= 9 \cdot \frac{1}{3} x^{-2/3} = 3x^{-2/3} \\
g''(x) &= 3 \cdot \frac{-2}{3} x^{-5/3} = -2x^{-5/3}
\end{aligned}
$$

(b) g'' is nonzero everywhere and undefined at $x = 0$. For $x < 0$, $g''(x) > 0$. For $x > 0$, $g''(x) < 0$. So g is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$.

(c) Concavity changes at $(0, 4)$ as we said in part (b).

5. We haven't covered this yet.