Discussion Problems for Math 180

Thursday, January 22, 2015

Review

1. Using what you know about trigonometry, calculate $\sin\left(\frac{\pi}{12}\right)$.

2. Locate the vertex of the parabola $y = x^2 - 4x$, and sketch its graph.

3. Express $\dfrac{1-\frac{2}{3}}{\frac{4}{5}+6}$ as a single fraction.

This time

4. (a) What, precisely, does it mean for a function f to be continuous?
   (b) Consider the family of functions

$$f_a(x) = \begin{cases} x^2 + a & \text{if } x < 0, \\ 1 - ax & \text{if } x \geq 0. \end{cases}$$

   Which of the functions f_a, if any, are continuous?

5. A piece of masonry falling from a tall building has height $h(t) = 80 - 5t^2$ above the ground at time t.
   (a) At what time does the falling debris hit the ground?
   (b) What is its height at $t = 2$?
   (c) What is its average speed between $t = 2$ and $t = 3$?
   (d) ... between $t = 2$ and $t = 2 + h$?
   (e) Check that your answer to part (d) agrees with your answer to part (c).
   (f) Consider your answer to part (d). It gives the average speed at which the debris falls from $t = 2$ to a time h seconds later. By taking the limit as $h \to 0$, we can determine the exact (instantaneous) speed with which the debris is falling at $t = 2$. What is this speed?

6. Compute the limit

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$