Discussion Problems for Math 180

Thursday, March 5, 2015

Review

1. If $f(x) = \tan^{-1}(3x - 2)$, what is $f^{-1}(x)$?
2. Find the first and second derivatives of each function:
   (a) $5\sin(x) - 4\cos(x)$
   (b) xe^{-x^2}
   (c) $\tan^{-1}(x)$
   (d) $\dfrac{2x-3}{x-5}$
3. For what positive value of x is x^x the smallest?
4. Prove that $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$ using implicit differentiation.

This time

5. On which intervals is xe^{-x^2} increasing? Decreasing?
6. On which intervals are the following functions concave up? Concave down?
   (a) $x^4 - 2x^3 + 1$
   (b) $\dfrac{2x-3}{x-5}$
7. Sketch the graph of a differentiable function $f(x)$ on $(-\infty, 0) \cup (0, \infty)$ such that $f'(x) < 0$ for $x < 0$, $f'(x) > 0$ for $x > 0$, and $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 2$.
8. A patient comes into the emergency room with a myocardial infarction. You administer nitroglycerin as a vasodilator, causing the radii of the blood vessels to increase by 2% per minute. The Hagen-Poiseuille equation from fluid dynamics tells us that the blood flow through a vessel is directly proportional to the fourth power of its radius. The flow must increase by at least 10% per minute or your patient will die. What happens?

   (Hints: You're given r'/r and need to find Q'/Q, where Q is the blood flow. You know that $Q = kr^4$ for some constant k. Don't worry about whether you've seen a problem like this before, just start doing the calculations and see what happens. Guessing whether the patient lives or dies without any support will not earn much credit.)