Discussion Problems for Math 180

Tuesday, April 28, 2015

Review answers

- Definitions:
  - A function f is *continuous* at $x = a$ if $\lim_{x \to a} f(x) = f(a)$.
  - The *derivative* of a function f at a point a is the number $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.
  - The function g is the *inverse function* of f if we have $f(x) = y$ whenever $g(y) = x$.
- Basic properties of exponential and logarithmic functions:

$$e^{\ln(x)} = x \qquad \ln(e^x) = x$$

$$e^a \cdot e^b = e^{a+b} \qquad \ln(xy) = \ln(x) + \ln(y)$$

$$(e^a)^n = e^{an} \qquad \ln(x^n) = n\ln(x)$$

$$\frac{e^a}{e^b} = e^{a-b} \qquad \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y) \qquad \text{[optional]}$$

- Derivative rules:

Additivity: $[f(x) + g(x)]' = f'(x) + g'(x)$

Scaling: $[c\,f(x)]' = cf'(x)$

Chain rule: $[f(g(x)]' = f'(g(x))g'(x)$

Product rule: $[f(x)\,g(x)]' = f'(x)\,g(x) + f(x)\,g'(x)$

Quotient rule: $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

This time

1. Take derivatives:

   (a) $x^3 - 4x + 3$

   (b) $x\sin(x) - \cos(x)$

   (c) $\tan^{-1}(e^x - 1)$

   (d) $\dfrac{1}{\sqrt{2\pi}}e^{-x^2/2}$

   (e) 4^x

   (f) $x\left(40 - \dfrac{1}{x}\right)$

2. Take limits:

(a) $\lim_{x\to 0} \frac{\sin(x)}{x+3}$

(b) $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

(c) $\lim_{x\to\infty} \frac{\cos(x)}{x^2+7}$

(d) $\lim_{s\to 2} \frac{s^2 - s - 2}{s^2 - 5s + 6}$

(e) $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right)$

3. Prove, directly from the definition of the derivative, that

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}.$$

4. Prove that

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{x^2+1}.$$

You may use the fact that $\frac{d}{dx}\tan(x) = \sec^2(x)$.

5. Use linear approximation to approximate $\sqrt[3]{65}$. Indicate which function you are using and at which point you are expanding, and determine (with justification) whether you've obtained an over- or underestimate.

6. Sketch a graph of the function $f(x) = x^4 - 6x^2 + 5$. (Indicate monotonicity, concavity, asymptotes, end behavior, etc., providing full justification.)

7. Find the point on the line $y = 1 - 2x$ closest to the point $(0,0)$.

8. Calculate integrals:

(a) $\int x^3 - 4x + 3\,dx$

(b) $\int \sin(x)\,dx$

(c) $\int e^{2x} - 2e^x + 3\,dx$

(d) $\int \frac{dx}{1-3x}$

(e) $\int \sin(x)^5 \cos(x)\,dx$

(f) $\int \sqrt{x}\,dx$

(g) $\int \frac{2\,dx}{x^2+4}$

(h) $\int_{-\pi}^{\pi} \cos(x)\,dx$

(i) $\int_{-2}^{2} e^{x^2+1}\sin(x)\,dx$

9. Write the equation of the tangent line to the curve

$$(x^2+y^2)^2 = x^3 - 3xy^2$$

at the point $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$.

Next time: Take the final exam from Fall 2013 (available on the course website) and bring in your solutions. We will go over them in class on Thursday.

The final will be held Thursday, May 7 from 1 PM – 3 PM in Lecture Center B101.