Practice Exam 1 Solutions

Tuesday, February 10, 2015

1. Fill in the blanks:

   (a) A function $f(x)$ is continuous at a point a if $\underline{\lim_{x\to a} f(x) = f(a)}$.

   (b) The derivative of a function $g(x)$ at a point b is $\underline{\lim_{h\to 0} \frac{g(b+h)-g(b)}{h}}$. (Other answers possible.)

   (c) The function $h(x)$ has a horizontal asymptote at $y = c$ if $\underline{\lim_{x\to\infty} h(x) = c}$ or $\underline{\lim_{x\to -\infty} h(x) = c}$.

   (d) The function $j(x)$ has a vertical asymptote at $x = d$ if $\underline{\lim_{x\to d^-} j(x) = \pm\infty}$ or $\underline{\lim_{x\to d^+} j(x) = \pm\infty}$.

2. Calculate:

   (a) $\lim_{x\to\infty} 2\arctan(x) = 2\left(\frac{\pi}{2}\right) = \pi.$

   (b) $\lim_{x\to\infty} \frac{x^2-4x+1}{(2x-1)^2} = \lim_{x\to\infty} \frac{x^2-4x+1}{4x^2-4x+1} = \lim_{x\to\infty} \frac{x^2-4x+1}{4x^2-4x+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1-\frac{4}{x}-\frac{1}{x^2}}{4-\frac{4}{x}+\frac{1}{x^2}} = \frac{1}{4}$

3. Calculate:

   (a) $\frac{d}{dx}\left[\frac{\sqrt{x}}{x^2-1}\right] = \frac{(x^2-1)\frac{1}{2\sqrt{x}} - \sqrt{x}(2x)}{(x^2-1)^2}$ (Remember, you don't have to simplify.)

   (b) $\frac{d}{dx} 3e^x \sin(x) = 3e^x\sin(x) + 3e^x\cos(x)$

   (c) $\frac{d}{dx}\left[\frac{e^2\pi - 1}{2}\right] = 0$

4. Use the definition of the derivative to calculate the derivative of the function $\frac{4}{2-3x}$.

$$f'(x) = \lim_{h\to 0} \frac{\frac{4}{2-3(x+h)} - \frac{4}{2-3x}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{4}{2-3(x+h)} - \frac{4}{2-3x}}{h} \cdot \frac{(2-3x-3h)(2-3x)}{(2-3x-3h)(2-3x)}$$
$$= \lim_{h\to 0} \frac{4(2-3x) - 4(2-3x-3h)}{h(2-3x-3h)(2-3x)} = \lim_{h\to 0} \frac{4(3h)}{h(2-3x-3h)(2-3x)}$$
$$= \lim_{h\to 0} \frac{4\cdot 3}{h(2-3x-3h)(2-3x)} = \frac{12}{(2-3x)^2}$$

5. Consider the function

$$f(x) = \begin{cases} 2 & \text{if your instructor likes the number } x, \\ 5 & \text{if your instructor does not like the number } x. \end{cases}$$

Determine the limit

$$\lim_{x\to 1^+} \frac{f(x)}{x-1}.$$

Justify your answer.

Clearly $f(x) \geq 2$, so

$$\frac{f(x)}{x-1} \geq \frac{2}{x-1}.$$

Now

$$\lim_{x\to 1^+} \frac{2}{x-1} = +\infty,$$

so by the squeeze theorem we have

$$\lim_{x\to 1^+} \frac{f(x)}{x-1} = +\infty.$$