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(1) $x^2+y^2-2x+6y-6=0$

$x^2-2x \quad +y^2+6y = 6$

$x^2-2x+1 + y^2+6y+9 = 6+1+9=16$

$(x-1)^2 + (y+3)^2 = 4^2$

$\boxed{\text{Center } (1,-3) \quad \text{Radius } 4}$

(2) $x^4-16 = (x^2-4)(x^2+4) = \boxed{(x-2)(x+2)(x^2+4)}$

– or – $\boxed{(x-2)(x+2)(x-2i)(x+2i)}$

(3) $\frac{d}{dx}\sqrt{x} = \lim\limits_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$ ☆

$= \lim\limits_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim\limits_{h\to 0} \frac{(x+h)-x}{h\left[\sqrt{x+h}+\sqrt{x}\right]}$

$= \lim\limits_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left[\sqrt{x+h}+\sqrt{x}\right]} = \lim\limits_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

☆ Note: Using l'Hôpital's rule here would be cheating, since that requires us to know $\frac{d}{dx}\sqrt{x}$ already.

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(5) (b) $\lim_{x\to\infty} x - \sqrt{x^2+4x} = \lim_{x\to\infty} x\left[1 - \frac{\sqrt{x^2+4x}}{x}\right]$

$= \lim_{x\to\infty} \frac{\underbrace{1-\sqrt{1+4/x}}_{\to 0}}{\underbrace{1/x}_{\to 0}}$

$\overset{\text{l'Hôp}}{=} \lim_{x\to\infty} \frac{-\frac{1}{2\sqrt{1+4/x}} \cdot 4\cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{\frac{-1}{x^2}}}$

$= \lim_{x\to\infty} \frac{-2}{\sqrt{1+4/x}} = \boxed{-2}$

(c) $\lim_{x\to 0^+} x^{2x} = \lim_{x\to 0^+} e^{2x\ln(x)} \overset{*}{=} \exp\left\{\lim_{x\to 0^+} 2x\ln(x)\right\}$

$= \exp\left\{\lim_{x\to 0^+} \frac{\overbrace{2\ln(x)}^{\to\infty}}{\underbrace{1/x}_{\to\infty}}\right\} \overset{\text{l'Hôp}}{=} \exp\left\{\lim_{x\to 0^+} \frac{2/x}{-1/x^2}\right\}$

$= \exp\left\{\lim_{x\to 0^+} \frac{-2x^2}{x}\right\} = \exp\left\{\lim_{x\to 0^+} -2y\right\} = e^0 = 1$

(6) $6 + (0.1)(-12) = 6 - 1.2 = 4.8$

$*$ Only true if the limit actually exists.