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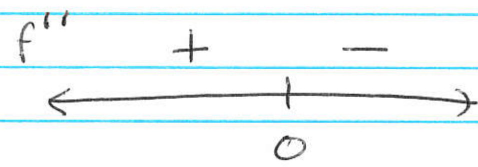
(1)

$$\begin{array}{l} 1 + 2 + 3 + \cdots + 1{,}000 + \\ 2{,}000 + 1{,}999 + 1{,}998 + \cdots + 1{,}001 \\ \hline \underbrace{2{,}001 + 2{,}001 + 2{,}001 + \cdots + 2{,}001}_{1{,}000 \text{ times}} \end{array}$$

$$(2{,}001)(1{,}000) = \boxed{2{,}001{,}000}$$

(2)

$$f(x) = \arctan(x)$$



$$f'(x) = \frac{1}{1+x^2}$$

$$f''(x) = \frac{-1}{(1+x^2)^2}(2x)$$

$$\boxed{\begin{array}{l} \text{concave up on } (-\infty, 0) \\ \text{concave down on } (0, \infty) \end{array}}$$

(3)

$$y = 4^x$$

$$\ln(y) = x \ln(4)$$

$$\frac{1}{y}\frac{dy}{dx} = \ln(4)$$

$$\frac{dy}{dx} = y \ln(4) = \boxed{4^x \ln(4)}$$

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(4)

(a) $x^2 - 3x + C$

(b) $-\cos(x) + C$

(c) $\arctan(x) + C$

(d) $\frac{4}{3}\ln(x) + C$

(e) $\frac{-6}{x} + C$

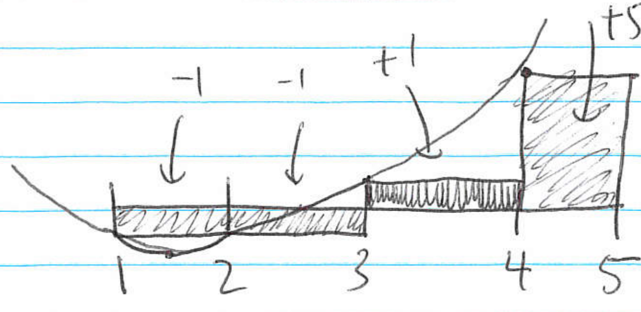
(5) $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$; $\frac{1}{2x^2+3} = \frac{1}{3}\left(\frac{1}{1+\frac{2}{3}x^2}\right)$

$\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}x\right) = \frac{1}{1+\left(\frac{\sqrt{2}}{\sqrt{3}}x\right)^2}\left(\frac{\sqrt{2}}{\sqrt{3}}\right) = \frac{1}{1+\frac{2}{3}x^2}\cdot\frac{\sqrt{2}}{\sqrt{3}}$

$\frac{d}{dx}\left(\frac{1}{3}\cdot\frac{\sqrt{3}}{\sqrt{2}}\cdot\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}x\right)\right) = \frac{1}{3}\left[\frac{1}{1+\frac{2}{3}x^2}\right] = \frac{1}{3+2x^2}$

$$\boxed{\frac{\sqrt{3}}{3\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}x\right) + C}$$

(6)



$f(1) = -1$
$f(2) = -1$
$f(3) = 1$
$f(4) = 5$

$(-1)+(-1)+1+5 = \boxed{4}$

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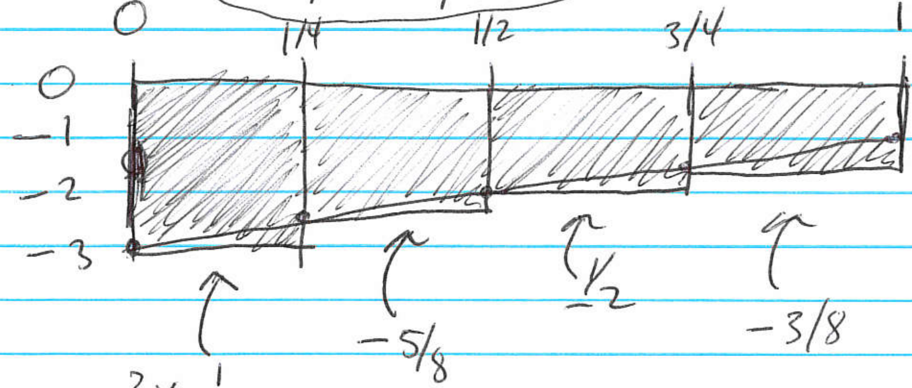
7 a



$f(0) = -3$
$f(1/4) = -5/2$
$f(1/2) = -2$
$f(3/4) = -3/2$

$-3 \times \frac{1}{4}$

$= -3/4$

$$-\frac{3}{4} - \frac{5}{8} - \frac{1}{2} - \frac{3}{8} = \frac{-18}{8} = \boxed{-\frac{9}{4}}$$

b $\frac{1}{n}\left[(2(0)-3) + \left(2\left(\frac{1}{n}\right)-3\right) + \left(2\left(\frac{2}{n}\right)-3\right) + \cdots + \left(2\left(\frac{n-1}{n}\right)-3\right)\right]$

c $\frac{1}{4}\left[(2(0)-3) + (2(1/4)-3) + (2(1/2)-3) + (2(3/4)-3)\right]$

$= \frac{1}{4}\left[(-3) + (-5/2) + (-2) + (-3/2)\right]$

$= \frac{1}{4}(-9) = \boxed{\frac{-9}{4}}$

d $\frac{1}{n}\left[2\left(0 + \frac{1}{n} + \frac{2}{n} + \cdots + \frac{n-1}{n}\right) - 3n\right]$

$= \frac{1}{n}\left[\frac{2}{n}\left[0+1+\cdots+(n-1)\right] - 3n\right] = \frac{2}{n^2}\left[0+1+\cdots+(n-1)\right] - 3$

$= \frac{2}{n^2} \frac{(n-1)n}{2} - 3 = \boxed{\frac{(n-1)}{n} - 3}$

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Since we saw

$$1 + \cdots + n = \frac{n(n+1)}{2}$$

in problem 1.

ⓔ $$\lim_{n\to\infty} \frac{n-1}{n} - 3 = \lim_{n\to\infty} 1 - \underbrace{\frac{1}{n}}_{\to 0} - 3$$

$$= 1 - 3 = \boxed{-2}$$