4/14/15

① ⓐ $f(x)=\sqrt{x}$ Approx at $x=9$ $f(9)=3$

$f'(x)=\frac{1}{2\sqrt{x}}$ $f'(9)=\frac{1}{6}$

$$f(x)\approx 3+\frac{1}{6}(x-9)\ ;\ f(8)\approx 3+\frac{1}{6}(8-9)=\boxed{2+5/6}$$

ⓑ $f(x)=\frac{8}{\sqrt{x}}$ Approx at $x=9$ $f(9)=\frac{8}{3}$

$f'(x)=8\left(\frac{-1}{2}x^{-3/2}\right)=-4x^{-3/2}$ $f'(9)=-4(9)^{-3/2}=\frac{-4}{27}$

$$f(x)\approx\frac{8}{3}-\frac{4}{27}(x-9)\ ;\ f(8)\approx\frac{8}{3}+\frac{4}{27}=\frac{76}{27}=\boxed{2+22/27}$$

ⓒ $f''(x)=\frac{1}{2}\left(\frac{-1}{2}x^{-3/2}\right)$

$=\frac{-1}{4}x^{-3/2}$

f concave down

$\Rightarrow$ $\boxed{\text{overestimate}}$

$g''(x)=-4\left(\frac{-3}{2}x^{-5/2}\right)$

$=6x^{-5/2}$

g concave up

$\Rightarrow$ $\boxed{\text{underestimate}}$

So $\boxed{2+22/27<\sqrt{8}<2+5/6}$

② $$\lim_{x\to0^+}\ln\left(\tan\left(x+\frac{\pi}{2}\right)\right)^x=\lim_{x\to0^+}e^{x\ln(\ln(\tan(x+\pi/2)))}=\exp\left\{\lim_{x\to0^+}\frac{\overset{\to\infty}{\ln(\ln(\tan(x+\pi/2)))}}{\underset{\to\infty}{\frac{1}{x}}}\right\}$$

$$\underset{\text{l'Hôp}}{=}\exp\left\{\lim_{x\to0^+}\frac{\frac{1}{\ln(\tan(x+\frac{\pi}{2}))}\cdot\frac{1}{\tan(x+\frac{\pi}{2})}\cdot\sec^2(x+\pi/2)}{-1/x^2}\right\}$$

$$=\exp\left\{\lim_{x\to0^+}\frac{-x^2\ \cancel{\cos(x+\pi/2)}}{\ln(\tan(x+\pi/2))\ \sin(x+\pi/2)\ \cos^2(x+\pi/2)}\right\}$$

$$=\exp\left\{\lim_{x\to0^+}\frac{-x^2}{\ln(\tan(x+\pi/2))\cos(x)(-\sin(x))}\right\}=\exp\left\{\lim_{x\to0^+}\overset{\to1}{\frac{x}{\sin(x)}}\cdot\lim_{x\to0^+}\overset{\to0}{\frac{x}{\underset{\to\infty}{\ln(\tan(x+\frac{\pi}{2}))}\underset{\to1}{\cos(x)}}}\right\}$$

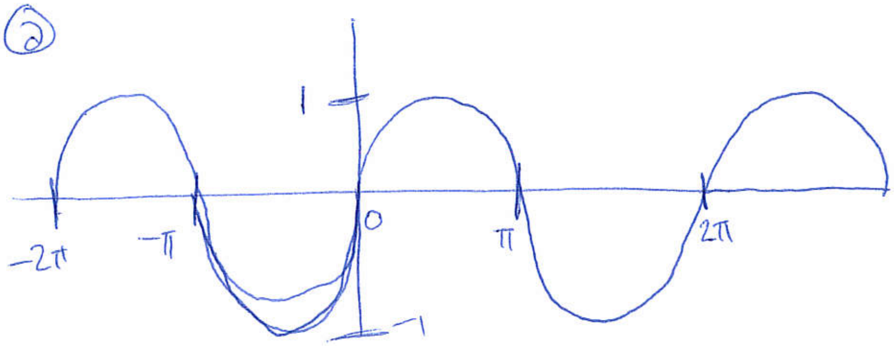
$$=\exp\{1\cdot0\}=e^0=\boxed{1}$$

Didn't mean for this to be so exciting, but you can do it!

4/4/15

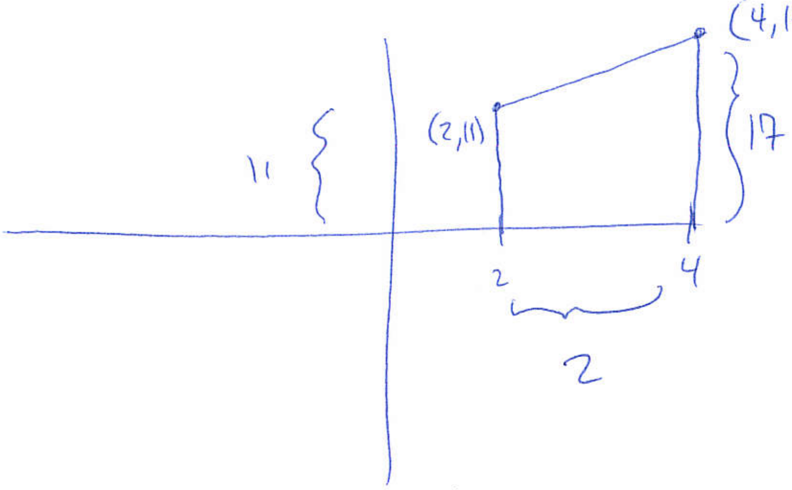
③ B and E.

④ ⓐ



ⓑ $\int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi} = [-(-1)] - (-1) = 1+1 = \boxed{2}$

⑤

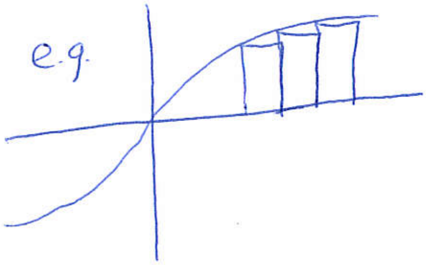


Trapezoid of area

$2\cdot\left(\frac{11+17}{2}\right) = \boxed{28}$

⑥ An underestimate, since arctan(x) is an increasing function:

e.g.



⑦ $\frac{d}{dx}\, x\cos(x) = -x\sin(x) + \cos(x)$

— Let's get rid of the sign. —

$\frac{d}{dx} -x\cos(x) = x\sin(x) - \cos(x)$

— Let's get rid of the cosine. —

$\frac{d}{dx} -x\cos(x) + \sin(x) = x\sin(x) - \cos(x) + \cos(x) = x\sin(x)$ ✓

$$\boxed{\int x\cos(x)\,dx = -x\cos(x) + \sin(x) + C}$$