4/21/15

1\. (a) f is continuous at a if $\lim_{x \to a} f(x) = f(a)$.

(b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

(c) $\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=0}^{n-1} f\left(a + i\,\frac{b-a}{n}\right) \cdot \left(\frac{b-a}{n}\right)$

(d) $\int_a^b F'(x)\,dx = F(b) - F(a)$

2\. $\ln(12) = \ln(2 \cdot 2 \cdot 3) = 2\ln(2) + \ln(3)$

$\Rightarrow \ln(3) = \ln(12) - 2\ln(2)$

$$\begin{array}{r} 0.6931 \\ \times \quad 2 \\ \hline 1.3862 \end{array} \qquad \begin{array}{r} 2.4849 \\ -1.3862 \\ \hline \boxed{1.0987} \end{array}$$

3\. $\frac{d}{dx}\left[\ln\left(\frac{x^3 - 5x + 1}{x^6 - 5x^4 + 3x - 2}\right)\right] = \frac{d}{dx}\left[\ln(x^3 - 5x + 1) - \ln(x^6 - 5x^4 + 3x - 2)\right]$

$= \frac{1}{x^3 - 5x + 1} \cdot (3x^2 - 5) - \frac{1}{x^6 - 5x^4 + 3x - 2} \cdot (6x^5 - 20x^3 + 3)$

4. (a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x)\Big|_0^1 = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

(b) $\displaystyle\int_0^1 \frac{dx}{1+x^2} = \tan^{-1}(x)\Big|_0^1 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

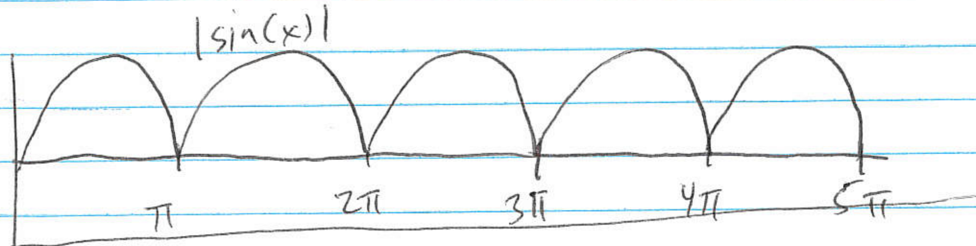
(c) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1+x}} = \int_1^2 \frac{du}{\sqrt{u}} = \Big[2\sqrt{u}\Big]_1^2 = 2\sqrt{2} - 2$

$$\boxed{\begin{array}{ll} u = 1+x & u(0) = 1 \\ du = dx & u(1) = 2 \end{array}}$$

(d) $\displaystyle\int_0^1 \frac{dx}{1+x} = \int_1^2 \frac{du}{u} = \ln(u)\Big|_1^2 = \ln(2) - \ln(1)$

$$\boxed{\begin{array}{ll} u = 1+x & u(0) = 1 \\ du = dx & u(1) = 2 \end{array}} \qquad = \boxed{\ln(2)}$$

5



$$\int_0^{5\pi} |\sin(x)|\,dx = 5\int_0^{\pi} \sin(x)\,dx$$

$$= 5\Big[-\cos(x)\Big]_0^{\pi} = 5\Big[1 - (-1)\Big] = \boxed{10}$$

4/21/15

(6) If $F'(x) = \ln(\cos(x))$, then

$$\frac{d}{dx}\int_{-\pi x/2}^{\pi x/2} \ln(\cos(t))\,dt = \frac{d}{dx}\left[F\left(\frac{\pi x}{2}\right) - F\left(\frac{-\pi x}{2}\right)\right]$$

$$= F'\left(\frac{\pi x}{2}\right)\cdot\left(\frac{\pi}{2}\right) - F'\left(\frac{-\pi x}{2}\right)\cdot\left(\frac{-\pi}{2}\right)$$

$$= \ln\left(\cos\left(\frac{\pi x}{2}\right)\right)\cdot\frac{\pi}{2} + \ln\left(\cos\left(\frac{-\pi x}{2}\right)\right)\cdot\frac{\pi}{2}$$

$$= \ln\left(\cos\left(\frac{\pi x}{2}\right)\right)\cdot\frac{\pi}{2} + \ln\left(\cos\left(\frac{\pi x}{2}\right)\right)\cdot\frac{\pi}{2}$$

$$= \boxed{\pi \ln\left(\cos\left(\frac{\pi x}{2}\right)\right)}$$