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① (a) $\boxed{3x^2 - 4}$

(b) $x\cos(x) + \sin(x)$
$+ \sin(x)$
$= \boxed{x\cos(x) + 2\sin(x)}$

(c) $\boxed{\dfrac{e^x}{(e^x-1)^2+1}}$

(d) $\boxed{\dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}(-x)}$

(e) $\dfrac{d}{dx} 4^x = \dfrac{d}{dx} e^{x\ln 4} = \boxed{4^x \ln(4)}$

(f) $x\left(40 - \frac{1}{x}\right) = 40x - 1$
$\boxed{40}$

② (a) $\lim\limits_{x\to 0} \dfrac{\sin(x)}{x+3} = \dfrac{\sin(0)}{0+3} = \boxed{0}$

(b) $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - x^2}{h} = \lim\limits_{h\to 0} \dfrac{x^2+2xh+h^2-x^2}{h}$

$= \lim\limits_{h\to 0} \dfrac{2xh-h^2}{h} = \lim\limits_{h\to 0} 2x-h = \boxed{2x}$

(c) $\lim\limits_{x\to\infty} \dfrac{\cos(x)}{x^2+7} = \boxed{0}$

bounded by ±1

$\to +\infty$

(d) $\lim\limits_{s\to 2} \dfrac{s^2-s-2}{s^2-5s+6} = \lim\limits_{s\to 2} \dfrac{(s-2)(s+1)}{(s-2)(s-3)}$

$= \lim\limits_{s\to 2} \dfrac{s+1}{s-3} = \dfrac{3}{-1} = \boxed{-3}$

(e) $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right)$

$-1 \le \sin\left(\frac{1}{x^2}\right) \le 1$
$-x^2 \le x^2\sin\left(\frac{1}{x^2}\right) \le x^2$

$\lim\limits_{x\to 0} -x^2 = \lim\limits_{x\to 0} x^2 = 0$, so

by the squeeze theorem

$\boxed{\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right) = 0}$

③ $\dfrac{d}{dx}\sqrt{x} = \lim\limits_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h} = \lim\limits_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h} \dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim\limits_{h\to 0} \dfrac{(x+h)-x}{h\left[\sqrt{x+h}+\sqrt{x}\right]}$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}}{\cancel{h}\left[\sqrt{x+h}+\sqrt{x}\right]} = \lim\limits_{h\to 0} \dfrac{1}{\sqrt{x+h}+\sqrt{x}} = \boxed{\dfrac{1}{2\sqrt{x}}}$

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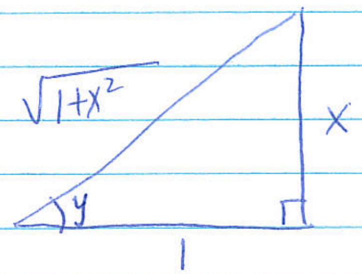
(4) If $y = \tan^{-1}(x)$ then $x = \tan(y)$, so

$$1 = \sec^2(y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \cos^2(y) = \cos^2(\tan^{-1}(x))$$



$\tan(y) = x$

$\cos(y) = \dfrac{1}{\sqrt{1+x^2}}$

$$\Rightarrow \boxed{\frac{d}{dx}\tan(x) = \frac{1}{1+x^2}}$$

(5) Take $f(x) = \sqrt[3]{x}$, $a = 64$.

$$f'(x) = \frac{1}{3}x^{-2/3}$$

$$\sqrt[3]{x} \approx \sqrt[3]{64} + \frac{1}{3(\sqrt[3]{64})^2}(x-64)$$

$$= 4 + \frac{1}{48}(x-64)$$

$$\boxed{\sqrt[3]{65} \approx 4 + 1/48}$$

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(6)

$$f(x) = x^4 - 6x^2 + 5 = (x^2-1)(x^2-5) = (x-1)(x+1)(x-\sqrt{5})(x+\sqrt{5})$$

$$f'(x) = 4x^3 - 12x = 4x(x^2-3) = 4x(x-\sqrt{3})(x+\sqrt{3})$$

$$f''(x) = 12x^2 - 12 = 12(x^2-1) = 12(x+1)(x-1)$$

f': $\ominus$ $-\sqrt{3}$ $\oplus$ 0 $\ominus$ $\sqrt{3}$ $\oplus$

f'': $\oplus$ -1 $\ominus$ 1 $\oplus$

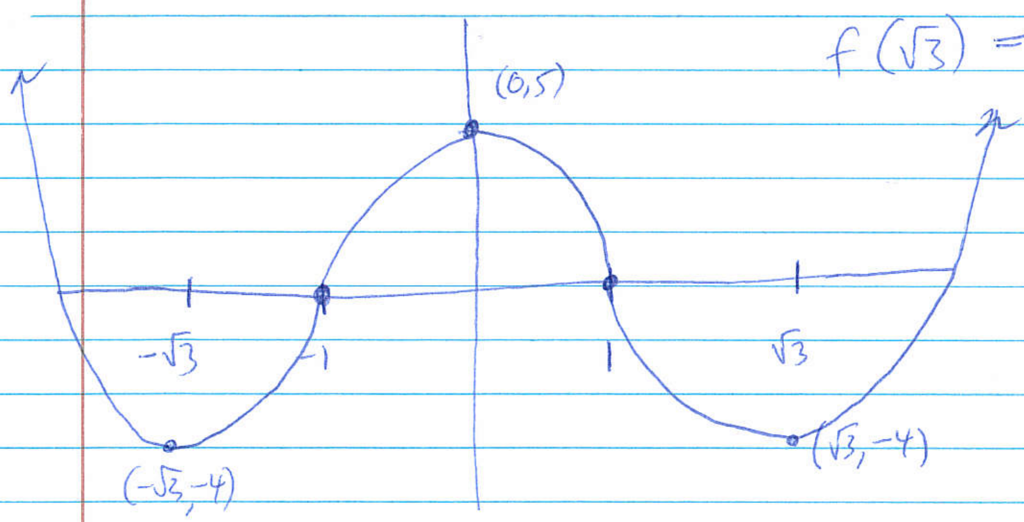
$f(-\sqrt{3}) = 9 - 18 + 5 = -4$

$f(-1) = 0$

$f(0) = 5$

$f(1) = 0$

$f(\sqrt{3}) = -4$



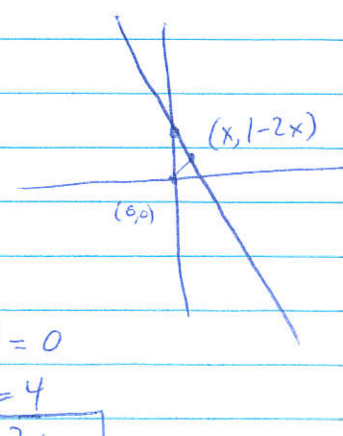
$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

(7) Want to minimize $\sqrt{x^2+(1-2x)^2}$.

May as well minimize $x^2 + (1-2x)^2$

$$= x^2 + 1 - 4x + 4x^2$$
$$= 5x^2 - 4x + 1$$



$f'(x) = 10x - 4$

$f''(x) = 10 \Rightarrow$ always concave up

$10x - 4 = 0$

$10x = 4$

$\boxed{x = 2/5}$

(8) (a) $\int x^3 - 4x + 3 \, dx$

$= \boxed{\frac{1}{4}x^4 - 2x^2 + 3x + C}$

(b) $\boxed{-\cos(x) + C}$

(c) $\boxed{\frac{1}{2}e^{2x} - 2e^x + 3x + C}$

(d) $\int \frac{dx}{1-3x} = \int \frac{-\frac{1}{3}\,du}{u} = \frac{-1}{3}\ln(u)$

$u = 1-3x$
$-3\,dx = du$
$\Rightarrow dx = \frac{-1}{3}du$

$= \boxed{\frac{-1}{3}\ln(1-3x)}$

(e) $\int \sin(x)^5 \cos(x)\,dx = \int u^5\,du = \frac{u^6}{6} + C$

$u = \sin(x)$
$du = \cos(x)\,dx$

$= \boxed{\frac{\sin(x)^6}{6} + C}$

(f) $\int x^{1/2}\,dx = \boxed{\frac{x^{3/2}}{3/2} + C}$

(g) $\int \frac{2\,dx}{x^2+4} = 2\int \frac{dx}{x^2+4} = \frac{2}{4}\int \frac{dx}{\frac{x^2}{4}+1}$

$= \frac{1}{\cancel{2}}\int \frac{\cancel{2}\,du}{u^2+1} = \tan^{-1}(u) + C$

$u = x/2$
$du = dx/2$
$\Rightarrow dx = 2du$

$= \boxed{\tan^{-1}\left(\frac{x}{2}\right)}$

(h) $\int_{-\pi}^{\pi} \cos(x)\,dx = \sin(x)\Big|_{-\pi}^{\pi}$

$= \sin(\pi) - \sin(-\pi) = 1 - (-1) = \boxed{2}$

(i) $\int_{-2}^{2} e^{x^2+1}\sin(x)\,dx = \boxed{0}$

Since this function is odd.

(9) $(x^2+y^2)^2 = x^3 - 3xy^2$

$2(x^2+y^2)(2x+2yy') = 3x^2 - 3y^2 - 6xyy'$

$2\left(\frac{1}{4}+\frac{3}{4}\right)\left(2\left(\frac{-1}{2}\right)+2\left(\frac{\sqrt{3}}{2}\right)y'\right) = 3\left(\frac{1}{4}\right) - 3\left(\frac{3}{4}\right) + 6\left(\frac{-1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)y'$

$-2 + 2\sqrt{3}y' = \frac{3}{2} - \frac{3\sqrt{3}}{2}y'$

$\frac{7\sqrt{3}}{2}y' = \frac{7}{2} \quad \Rightarrow y' = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\boxed{y - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}\left(x + \frac{1}{2}\right)}$