Newsletter

Mathematicians and Education Reform ...Forum...

Spring 2000 Volume 12 Number 2

Abandoning dead ends:

Resuscitating the Heart of Mathematics

by Edward B. Burger, Williams College and
Michael Starbird, University of Texas, Austin

> **Question to typical college graduate majoring in the liberal arts:** *You graduated from college 15 years ago. What was the final mathematics course you took?*
>
> **Former student:** *Pre-calculus.*
>
> **Interviewer:** *What was your final literature course?*
>
> **Former student:** *Pre-Shakespeare.*

Pre-calculus should never be a final mathematics course.
Pre-calculus and algebra provide essential techniques for a student who requires those specific skills in a technical profession or in subsequent courses. However, technical facility with manipulating algebraic expressions is soon forgotten by anybody who does not practice such manipulations. Moreover, algebra is not a used or useful life skill for most people.

Having students end their mathematical education with college algebra or pre-calculus perhaps is the result of a hierarchical view of mathematics. In the eyes of many, mathematics is an edifice built on a foundation that includes algebra and pre-calculus. Students often see mathematics as an unending string of courses that starts with arithmetic and progresses relentlessly through high school algebra, geometry, pre-calculus, calculus, and so on *ad infinitum* or perhaps *ad nauseum*. Each subsequent course is viewed as dependent on the previous one, and there is no significant independent payoff for a course except in taking the next step in the weary journey. This curricular paradigm results in students traveling along this road as far as they go and wherever they stop, their mathematical education ends and is quickly forgotten. If students stop prematurely (as all liberal arts majors will), then they summarize their mathematical education as, "I got as far as pre-calculus."

> **Question to typical college graduate majoring in the liberal arts:** *How would you describe the mathematics component of your college education?*
>
> **Good answer:** *Important, mind-opening, surprising, interesting, full of useful techniques of effective thinking, life-changing, culturally significant, educationally central.*
>
> **Actual answer:** *I completed my math requirement.*
>
> **Follow-up question to likely answer:** *What did you learn?*
>
> **Follow-up answer:** *I got a C.*

We have a tradition of letting students end their mathe-

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inside

from the editor

In *Resuscitating the Heart of Mathematics*, page 1, Edward Burger, Williams College, and Michael Starbird, University of Texas, Austin, admonish us that, "Mathematics has a great story to tell [and] we should not be forgiven for making mathematics appear mundane and boring." Their tremendous enthusiasm for spreading mathematics among the "masses," i.e., those students majoring in liberal arts and not oriented to mathematics or science, should inspire faculty to join them in their mission. A follow-up article will give details of their experiences in selecting mathematical ideas for a liberal arts math-requirement course— what worked and what bombed— and getting students to think deeply about the ideas.

In the Fall issue of the MER Newsletter, Paul Sally, University of Chicago, wrote about how a problem Marjorie Enneking, Portland State University, had presented at an MER workshop led to mathematical and teaching explorations. In *"An Open-Ended Problem on the Geoboard"* on page 3, Enneking fills in the history of how she first encountered the problem of finding all squares on a 6 x 6 geoboard, and the experiences she has had over years of introducing this problem to students and faculty. Students are inclined to keep to the technology of the geoboard (stretching rubber bands across pegs on a square grid), while faculty translate the problem into mathematical language. Enneking finds that, in general, the students are more successful!

The Rensselaer masters program in natural sciences, which Lester Rubenfeld describes in the article on page 8, is a masters program for high school mathematics and science teachers but with a difference. Steeped in a metacognitive approach, teachers delve into what it is to do science and mathematics as they deepen their experiences in teaching, especially considering inter-disciplinary approaches and the possibilities offered by new technologies.

Solomon Friedberg, Boston College, writes about the Boston College Mathematics Case Studies Project, page 12, as a new resource for helping graduate students develop their teaching skills as teaching assistants. Taking a leaf from professional training practices in business and law, the case studies of scenarios in undergraduate mathematics classes present true-to-life teaching dilemmas. Through group discussions, graduate students can analyze the situations objectively and consider a variety of strategies to resolve the problems.

Naomi D. Fisher

The **Mathematicians and Education Reform (MER) Forum** seeks the effective participation of mathematicians in mathematics education reform at the K-12, undergraduate, and graduate levels, and the recognition of the importance of these efforts to the well being of the mathematics community. The MER Forum envisages the pursuit of educational reform through informed discussion of educational issues, thoughtful responses to changing educational conditions, and the promotion of exemplary programs. The creation and support of a network of mathematicians with a sustained commitment to mathematics education is central to this vision.

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Notes on Doing/Teaching Mathematics

"An Open-Ended Problem on the Geoboard"

by Marjorie Enneking, Portland State University

I thoroughly enjoyed Paul Sally's article in the Fall, 1999 MER Newsletter about the geoboard problem. I thought people might be interested in a little more background on the problem and I also want to be sure that William J. Masalski, my original source, gets credit for this lovely problem. I originally saw it in an article titled "An Open-Ended Problem on the Geoboard" in the March, 1974, issue of The Mathematics Teacher. Bill's initial question was "How many squares of different sizes can you make on a 6 x 6 geoboard?"

We happened to have an old set of home-made 6 x 6 geoboards (plywood with nails) in the math lab, so I decided to ask my students this question. Just as Bill described, the students quickly came up with the obvious squares, the ones with horizontal and vertical sides, and those with sides at a 45-degree angle to the nail rows and columns. Several found more squares, more skewed, as they described them. After some discussion to determine that each figure really was a square, they were able to find the area of each square using the Pythagorean Theorem, Pick's Theorem, or some sort of dissection method. The students were quite happy with their results, organized them carefully to show that they had all possible cases, and thought they were done with the problem.

I might have been satisfied, too, except that Bill's article showed we had barely started. So I encouraged the students to find more squares and asked them if they were making any assumptions they didn't need to make. I was ready to rephrase the question to ask, "Using rubber bands, how many squares of different sizes can you make?" – with emphasis on the "s" in "bands" or to suggest that each group just start placing rubber bands on the geoboard and look hard for something surprising. But suddenly one group, who were using multiple rubber bands on the same geoboard to show the squares they had discovered thus far, suddenly noticed something. "Look at this, a square with only two vertices on the nails!" After reproducing the strange square on their own geoboards, the class discussed whether it was really a square and how to determine its size.

Now, of course, they were off and running-- finding squares with zero, one or two vertices on the nails. They got quite proficient at determining the measure of the side of these squares by finding the length of the "extended" edge between the closest nails on which the rubber band was anchored. By pushing together their geoboards in order to be able to place multiple lines cutting across the extended edge and perpendicular to it, they could divide the extended edge into equal parts and count how many parts were in the desired edge of the square.

Every time I have used this problem people have enjoyed working on it. Because they are so intrigued with the problem, even those with no particular love for mathematics find that some of those things they "had to learn" in high school are suddenly pretty handy to know. For example, in trying to convince others that their figure really is a square and not a rhombus, we usually wind up talking about slopes of lines and in particular, relative slopes of perpendicular lines. Occasionally someone will mutter, "Humph, I guess some of that stuff is useful after all."

I encourage you to look Masalski's article, but not too soon. You see, he includes sketches of the 56 different squares he had found at that time. And if you look at the article before trying it yourself, you will miss all the fun. An interesting sidelight of this problem, is that prospective elementary teachers often do better than mathematics majors or graduate students, and usually do much better than mathematicians. Why? I'm not sure, but I can tell you what I've noticed. Prospective elementary teachers are willing to experiment. They use the geoboards, play around with it, and watch for interesting things. It seems that the more mathematics background people have, the less likely are they to do that. Instead, they generally set the geoboard aside and reach for pencil and paper. They tend to try to approach it very analytically. And they usually have a very frustrating time and get nowhere, while students in a class for elementary teachers have done some rather nice work. They notice new and surprising figures, sort and classify them, try to determine if they have an exhaustive list, and draw on long forgotten knowledge from algebra and geometry to make convincing arguments.

I once heard a middle school teacher comment that she was the only mathematician at her school. Someone asked, "You mean you are the only mathematics teacher in your school?!" "Oh, no," she explained, "there are others in my school who teach math, but I'm the only one who does math." I decided then and there that I liked her definition of a mathematician - one who does math.

I hope you have fun with this problem, and thanks again to William Masalski for introducing me to it. ■

Resuscitating the Heart of Mathematics, *continued from page 1*

matics education without seeing truly enticing mathematical notions. Why do we persist in losing golden opportunities to bring intriguing, deep, and valuable ideas from mathematics to students? Most students proceed through a lock-step hierarchical program until they fall off the train far short of the ideas that we would describe as the mathematical highlands. The mathematical community has not made a thoughtful policy decision regarding this issue. Instead, inertia has played the dominant role in this curricular habit.

Things that are not used are not useful to the non-user. Mathematical methods such as linear programming and matrix manipulation are applied in the real world. However, does it follow that it is important to teach such techniques to people who will not directly use them? Of course, we should not go out of our way to avoid real applications of mathematics. However, the question for students having non-technical majors is whether mathematics can be of personal value to them. The argument that everyone might find uses for algebraic skills in actual life (to figure how to amortize a loan, for example) is extremely weak. Let's be honest with ourselves. How often in our everyday lives do we need to find the foci of a hyperbola? How often does anyone in a non-technical profession multiply matrices?

Question to typical college graduate majoring in the liberal arts: *What is mathematics?*

Student: *Mathematics is problems.*

Interviewer: *Do you like problems?*

Student: *No.*

Interviewer: *What is mathematics good for?*

Student: *Math gives the formulas you use to do problems at the end of each section of the textbook.*

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Question to a typical mathematics teacher: *What do you teach your students?*

Teacher: *I teach them how to solve problems.*

Interviewer: *Where do you get these problems?*

Teacher: *At the end of each section of the textbook.*

Mathematical codependency.

Students have clear expectations as they enter a mathematics course. They expect to have homework consisting of problems at the ends of sections of the textbook. They expect those problems to be identical to the problems that are worked out in the section except with the numbers changed. They expect tests consisting of problems similar to ones at the ends of sections with the numbers changed once again. Most teachers follow this model of teaching.

There is nothing wrong with this model if the goal is to have students mimic worked-out exercises from textbook sections. When we teach liberal arts students, we need to think honestly about what we hope students will take with them from the course and then emphasize those lessons in our teaching. We can highlight methods of thought that arise within mathematics and that are valuable to students as they solve problems not in textbook sections, but in their real, complex lives.

A vision for what mathematics can do—transform lives.

Mathematics courses *can* touch the lives of our students. Let's not have modest goals. We want our students to look at their lives, their habits of thought, and their world in a new and deeper way. That is the goal of the whole of education, and mathematics can play a central role in allowing all students to grow intellectually. All teachers should share the goal of improving the lives of their students—that is the purpose of education. However, it seems that the mathematics community has adopted a rather limited self-image about how profoundly our courses can affect students.

Mathematics involves penetrating techniques of thought that all people can use to solve problems, analyze situations, and sharpen the way they look at their world. Our mathematics courses should emphasize basic strategies of thought and analysis. Part of the power of mathematics lies in its inexorable quest for elegance, symmetry, order, and grace. Seeking pattern, order, and understanding is a transforming process that mathematics can help students develop. These and other strategies of thinking that have led to great ideas in mathematics can have their greatest value to people in making real-life decisions and facing situations that are completely outside mathematics. It is a crime for any student to leave a mathematics course with the impression that mathematics is a collection of mindless, rote procedures.

Stressing some basic "life lessons," inspired by mathematical thinking, can empower students to better grapple with and conquer the problems and issues that they face in their lives from love to business, from art to politics. If such basic, effective strategies of thought allow students to conquer infinity and the fourth dimension, then what can't they do? By emphasizing the process by which mathematicians create and discover concepts, we can find powerful strategies of thinking that are effective everywhere.

Our Top 10 Lessons for Life

1. Just do it.
2. Make mistakes and fail, but never give up.
3. Keep an open mind.
4. Explore the consequences of ideas.
5. Seek the essential.
6. Understand the issue.
7. Understand simple things deeply.
8. Break a difficult problem into easier ones.
9. Examine issues from several points of view.
10. Look for patterns and similarities.

Effective thinking surely is the main goal of any mathematics course. We want students to explore some of the greatest and most intriguing creations of human thought. Students must always realize that their primary job is to *think* and to learn methods of thinking that are illustrated in mathematics.

Coming to grips with hard ideas is not smooth sailing. Sometimes students will confront issues that start beyond their grasp, but our challenge is to teach students how to make vagueness turn into clarity and confusion evolve into comprehension. The journey to true understanding can be difficult and frustrating, but if we can engineer a happy ending, the challenges of that journey will be one of the highlights of the course.

Learning to fail.

Mathematics is not to be viewed from afar. Students should personally engage in the thinking of mathematical ideas. Somehow, we must invite our students to answer questions and not be afraid to make mistakes. Failing is the only way to learn. It is much better to guess a wrong answer than not to think about the question at all. But that idea is one that requires acculturation and must be built into the method of instruction. It is not in keeping with most of their mathematical experience. In short, failure must be encouraged.

Our wish is for students to be active participants in their own education. One goal is to bring students to see themselves as the discoverers of each idea. We want students to look beyond the mathematical concepts and not be satisfied with mere knowledge, but also challenge themselves to attain the power to figure things out on their own. When we present the idea of infinity, we must expose the thinking process by which a systematic exploration of a simple everyday concept of equality (that is, one-to-one correspondence) leads to an intellectual triumph. The idea of exploring consequences of clearly stated simple ideas is a lesson that can potentially last longer than the recollection of Cantor's diagonalization.

Mathematics has a great story to tell and we should be forgiven for any excesses of unbridled enthusiasm as we open doors for students to enter into worlds of profound interest and intrigue, but we should not be forgiven for making mathematics appear mundane and boring. This journey of the imagination and the mind should be fun. If students don't enjoy the course, then their thinking will end at the end of the final. Knowledge comes and goes, but hatred lasts forever. Creating a positive attitude for a lifetime has a continued, incrementally valuable effect. It influences not only the students in the class now, but it influences their presentation of a mathematical attitude to their children in the future. If either the teacher of the class or the students don't enjoy the experience, the class has not realized its true potential. Students should leave the class with the impression that there is much more fascination remaining to be discovered and learned. Let's leave them with a thirst for more.

Great ideas and great fun.

Mathematics contains some of the greatest ideas of humankind- ideas comparable to the works of Shakespeare, Plato, and Michelangelo. Mathematical ideas help shape history, and they can add texture, beauty, and wonder to the lives of all students. Deep, fascinating concepts in mathematics can be presented authentically to liberal arts students. It is an exciting adventure for students and their instructors to grapple with notions of infinity, the fourth dimension, chaos, fractals, coincidences, and the random.

Shall any gazer with mortal eyes
Or any searcher know with mortal mind
Veil after veil will lift but there must be
Veil after veil behind.
—Sir Edwin Arnold

After implementing this new vision of what mathematics can be

Interviewer: *What was the biggest idea you learned in college?*

Former student: *Infinity.*

Interviewer: *What was the most mind-expanding concept you learned in college?*

Former student: *The fourth dimension.*

Interviewer: *What class in college most improved your ability to think?*

Former student: *My math class.*

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MER Forum Announcements ..

Special Summer Issue on Minority Mathematicians

A special issue of the MER Newsletter devoted *to Minority Mathematicians: Who is responsible?* will be published in the summer. Abstracts for the four articles, three of which are based on presentations at the 2000 special session on Mathematics and Education Reform at the January Joint Math Meeting in Washington, DC, follow:

Minority mathematicians: Who is responsible?
Raymond Johnson, University of Maryland, College Park

The under-representation of minorities in science, mathematics and engineering is frequently explained by pointing fingers: grad schools blame colleges who blame high schools who blame middle schools.... Some have taken the opposite view. They approach the issue at their level and devise programs that work (and these programs share remarkable similarities). This issue contains descriptions of some such programs.

In the introductory article, a historical overview of programs that have worked and could be adopted anywhere will be provided. The fact that they have not been adopted other places means, in the words of the philosopher Pogo, “We have met the enemy and it is us.”

Minorities are invisible in mathematics
Sylvia T. Bozeman, Spelman College and William Yslas Vélez, University of Arizona

This article will discuss the invisibility of minority mathematicians, especially African American and Chicano mathematicians. A partial abstract follows:

Chicano mathematicians are almost invisible. The number of Chicanos that receive PhDs in mathematics each year probably can be counted on one hand, with fingers left over. There are certainly economic factors at work that exclude us from participating in the educational endeavors in this country. However, in the almost thirty years that I have participated in the mathematical enterprise of this country, I have witnessed behaviors of mathematicians and mathematics departments that have contributed to our under-representation in mathematical careers. One of the more egregious is the lack of Chicano faculty at our research universities in the Southwest. In the late 70's, there were seven Chicano research mathematicians at our PhD granting universities in the Southwest. Today, I can count only five! We have gone backwards. If this trend continues, in ten years, there may be no Chicano Full Professors of mathematics at these institutions. The contributions of these few Chicano mathematicians to the education of the minority students is exemplary. The fact there are so few minority mathematicians at these institutions has negatively impacted the education of the minority population. This article addresses some of the behaviors and attitudes that I think have contributed to our under-representation.

Attracting Undergraduate Minorities To Mathematics
Etta Z. Falconer, Spelman College

Minorities are severely underrepresented in mathematics. In the past this was true at Spelman College, but now five percent of Spelman students are mathematics majors. A comprehensive approach to attracting African American women to the mathematics major and preparing them for graduate study in mathematics has been successful at Spelman. The Model Institutions for Excellence Program (MIE) supports curricular reform, retention activities, infrastructure development, and student research in science, engineering and mathematics. The Mathematics Department has utilized collaborative programs with science departments as well as programs designed specifically for mathematics to develop strategies for the recruitment, retention and preparation of students for graduate study and careers in mathematics and mathematics-related areas.

... news briefs ... updates ...

Increasing the Number of Minority PhDs in Mathematics
David Manderscheid, University of Iowa

This academic year 20% of the 90 graduate students in the Mathematics Department at the University of Iowa are US minorities from groups underrepresented in mathematics. In the August 28, 1998 issue of Science the University of Iowa is listed as tied for fourth in the granting of PhDs in Mathematics to minorities from underrepresented groups for the period 1992-1996. This listing is based on the NSF/NIH/NEH/USED/USDA Survey of Earned Doctorates for the period. This article discusses the experiences of the University of Iowa in the recruitment and development of minority graduate students. Ideas that work and that might be transportable to other institutions are emphasized.

Views of MER Advisors

What Education Issues Do We Need To Look At?

In a discussion with MER advisors at the Annual Joint Mathematics Meeting in Washington, DC, participants were asked to highlight educational issues they felt were important to bring to the attention of the mathematics community at this time. The variety of interests and issues sparked a lively discussion that led in many directions. Informal discussion with leaders in education in the mathematics community has always been important in providing guidance for MER activities, but the diversity of ideas and intense involvement of these advisors seemed particularly noteworthy and we would like to share some of those ideas with our readers.

International Community: Deb Hughes Hallett
Mathematical research has been an international activity for many decades. Mathematics education, however, is only just beginning to reap the benefits of an international perspective. Unlike in research, where the US is one of the world leaders, in education, the US has much to learn from other countries.

Algebra and the Future: Deb Hughes Hallett
Today's ninth graders can buy a calculator that costs less than their sneakers and which can do all the algebraic manipulations they will learn in Algebra I and II. The ready accessibility of Computer Algebra Systems (CAS) is likely to impact all of us in the near future.

Mathematics Courses for Humanities Students: Michael Starbird
Can we make mathematics viewed as something everyone should know? Can math courses compete with other Liberal Arts classes whose goal is to change the life experience of students? (see *Resuscitating the Heart of Mathematics*, page 1.)

The Mathematical Education of Underrepresented Minorities: Raymond Johnson
What is the role of the full mathematical community in the education of minorities? What is the changing role of the minority serving institutions?

The Mathematical Preparation of Teachers: Jim Lewis
If the mathematics community valued the preparation of teachers, *it* would improve.

Orientation of New Faculty: Amy Cohen
What should a math department tell new members of its faculty about their teaching responsibilities? We need to find ways to provide not only immediate orientation and indoctrination, but also ongoing mentoring and "socialization." What should go into a useful letter on a job applicant's teaching experience?

Undergraduate Curriculum Foundations: Bill Barker
What are the mathematical needs of the students in the partner disciplines and how do we address these needs in our mathematics programs?

Evolution of Mathematical Education Reform: Chris Stevens
What is the impact of the mathematics education reform movement on new faculty and vice versa? Debate should lead to deeper thinking.

Articulation with High Schools, Community Colleges, and Colleges and Universities: Gail Burrill
What happens in grades 10-15 to make learning beneficial to students? What should students have for the next level? What is the impact at all levels of high stakes K-12 testing?

Rensselaer's Summer Masters in Natural Sciences Program

by Lester Rubenfeld, Rensselaer Polytechnic Institute

Backgrouund *In the 1960s and 70s Rensselaer Polytechnic Institute, with support from the National Science Foundation, offered its MS in Natural Sciences program to K-12 teachers. After running for nineteen years, the program was discontinued when NSF severely reduced funding for K-12 initiatives. In the late 1980s, as we became more and more involved in pre-college outreach, it was clear that a university like Rensselaer had a unique contribution to make to our K-12 colleagues. Not only does Rensselaer have a world class reputation as a research institution with a focus on mathematics, science and engineering, but it had begun to establish a reputation of being in the forefront of integrating technologies into its educational programs. At about the same time we also began to think about establishing a Center for Initiatives in Pre-College Education. Started in 1995, the Center has as part of its mission the development of graduate programs for K-12 teachers. This was an opportune time to update the previously offered MS in Natural Sciences program to build on Rensselaer's current expertise and reputation in interactive, inquiry pedagogy, and use of cutting edge technologies. The program graduated its first class in the summer of 1997, and has now reached a steady state. It has attracted secondary school mathematics and science teachers from over twenty five states and several foreign countries.*

MS in Natural Sciences Program Overview

The program is designed to be completed in three successive summers. Each summer session is six weeks and teachers earn ten credits per summer. There are six core courses taken together by all teachers: three in Summer 1, two in Summer 2, and one in Summer 3. In addition, each teacher takes three prescribed courses in his or her specific discipline, one in Summer 2 and two in Summer 3. Between summer sessions, participants are networked through a server on Rensselaer's campus.

To understand the nature of the program, it's helpful to look at two difficulties we had in implementing the program at a university like Rensselaer. First, though the program is definitely a rigorous one, it is not constructed in the same way as our usual Masters programs for mathematics and science majors. Several university curriculum committees must pass on new programs, and they were concerned that we would be "dumbing down" a Rensselaer degree. After much discussion and debate, we overcame that concern by promoting a philosophy that set a <u>floor for admission</u> into the program, which may be different than that for our regular admits. But over the course of the program we would help the participants to "grow" academically to where we would like them to be. We have successfully followed this philosophy by finding instructors who (i) respect secondary school teachers and can communicate well with them; (ii) possess content expertise; (iii) can model interactive, inquiry pedagogy; and (iv) can integrate technology into their classroom experiences. If we cannot find such a person on our own campus, we look to other local universities, and in fact almost 50% of our courses are now taught by adjuncts who have had experience interacting with K-12 teachers.

Second, we had to confront the role of technology: Should we expose teachers to technologies that they would never have a chance of seeing and using in their home districts, or should we keep their comfort level intact? We decided on the former, and feel strongly that all teachers should see the world <u>as it in fact can be</u>, even though their realities might be different. One result that many of our graduates have told us is that they are becoming change agents in their school districts and are "forcing" their administrations to purchase equipment and spend money on staff development in ways that had never happened before.

The program objectives are:

- to inspire observant inquiry in our participants;
- to give science and mathematics immediacy and relevance so that teachers will make them compelling to the youngsters in their charge;
- to provide knowledge of how content integrates with other disciplines, and provide instruction in using tools such as modern educational technologies to enhance classroom activities; and
- to enhance the skills and knowledge of secondary school mathematics and science teachers with techniques, tools and a rich body of relevant scientific and mathematics content to highlight the relevance of science and mathematics for their students.

The program departs from traditional graduate programs for teachers in several ways:

- It is <u>not</u> a program in "teacher education." Indeed, the strong focus on science, mathematics, and technology attracted many of the teachers to the program.
- <u>All</u> teachers, independent of discipline, study together in six core courses. These courses provide a natural

environment for collaborative, interdisciplinary work. Some of the final course projects have interdisciplinary requirements.
- Several of the core courses emphasize the nature and processes of science and mathematics.
- The use of modern instructional technologies, including computer software, web access and design, and interactive multi-media curriculum design, is a theme in all three summers and in a curriculum development project. Teachers are asked to apply these techniques to their classroom experiences.

How the program works

Summer 1 In the first summer, all teachers take the same introductory courses*: Nature & Processes of Natural Science*, *Nature of the Mathematical Sciences*, and *Introduction To Instructional Technologies*. This experience sets the social and academic expectations for the program. The teachers get to know each other quite well, developing both professional and social friendships, which are reinforced by the living arrangements. Almost all teachers, including those who are local, live either in the dormitories on campus or together in off-campus housing. In the *Nature & Processes of Natural Science* course mathematics teachers experience what it means to do and think about science, while in the *Nature of the Mathematical Sciences* course science teachers are immersed in an environment where they are compelled to think deeply about what it means to do mathematics. This metacognitive process of "thinking about how you think about mathematics and science" is not easy for many participants. Often, this is the first time that they have ever been asked to think this way. The discomfort level is quite high during the first summer, but by the end of the program most teachers are very comfortable with their experiences and some try to transfer them to their own classes.

Summer 2 In the second summer, all teachers take two core courses, *Teaching With Technology* and *Studying Teaching and Learning*, and one discipline specific course in Biology, Chemistry, or Mathematics. The complete set of discipline specific courses in Biology is *Human Biology*, *Biomolecular Science*, and *Biochemical Science*; the courses in Chemistry are *Environmental Chemistry*, *Molecular Structure and Spectra*, and *Principles of Modern Chemical Analysis*; and the courses in Mathematics are *Mathematics of Discrete Processes*, *Dynamical Mathematics*, and *Geometry: constructions, theory and applications*.

Teaching with Technology has a strong focus on the use of the world wide web as a curriculum tool, in addition to providing an introduction to the development of multimedia curricula materials. *Studying Teaching and Learning* is taught by a developmental psychologist from a local university who has worked with us for many years on developing a research agenda focused on how teaching and learning mathematics and science is affected by a highly charged, technological classroom environment. Her objectives are to have participants reflect on their own pedagogical practices, diagnose their students' understanding and misunderstanding, and examine perspectives on learning offered by various cognitive disciplines. A course project focuses on comparing teaching and learning across a range of interactional teaching contexts. This course completes the metacognitive strand of the program, which is aimed at making participants think deeply about mathematics, science and pedagogy.

Summer 3 In the third summer, participants take the last of the core courses, *Curriculum of the Future*, and two discipline specific courses. *Curriculum of the Future* is a technology course that builds on the instructional technology and discipline-specific components of the program the teachers have studied thus far. Participants are taught to use a powerful multimedia development tool, and are assigned a final project to design, in groups, an interdisciplinary, technology-based unit that they can use in their own classes. Our objective here is not only to expose participants to an additional compelling computer application, but to have them think deeply about their own understanding of the content matter.

Where we are

The program has graduated 57 teachers, of whom 30 are in mathematics. Of the 61 currently continuing teachers, 30 are in mathematics. The new class of 27 teachers has 13 mathematics teachers.

To date we are attracting mostly mathematics and biology teachers, together with a few chemistry teachers. There are so few young physics teachers that when we opened up enrollment to this population there were very few takers. Because the number of interested chemistry teachers is also low, we plan to reevaluate this component of the program.

We have set forth worthy objectives in this program, and believe that for the most part we are achieving them. However, what is most important is that participants are exposed to a very high level of thinking and discourse, both about their teaching disciplines and the pedagogy and tools that they use in their own classrooms. Although it is difficult to determine what <u>really</u> will make a difference, one strategy is clear to us. To accomplish anything we must hold a mirror in front of teachers so that they see the realities of their worlds, but we must help them to see also how they might change the realities to better accomplish our common goals of educating the work force for the 21st century. ■

Resuscitating the Heart of Mathematics, *continued from page 5*

Implementing great alternatives.
Mathematics departments can take the obvious action required, namely establish rich courses that teach valuable, deep, and culturally significant ideas and effective thinking to all students. Having students struggle up the first two rungs of a 100-rung ladder that they will never climb is a curricular strategy born of habit rather than thought. Let's not have students' mathematical journeys end in dead ends.

Some discoveries in mathematics are of such grandeur that they deserve to be known because they represent triumphs of the human intellect. But these grand triumphs of human thought are grand partly because the strategies of investigation that led to those victories over ignorance are so potent. Let's share these jewels with our students and allow them to see the power and excitement of thinking both within mathematics and more importantly, beyond.

What to do:
- ♦ Develop inspirational courses that celebrate effective thinking and great ideas.
- ♦ Change requirements so students do not end their mathematics experience without grappling with truly fascinating high points of mathematics.

How to do it:
- ♦ Develop courses that have your own stamp on them—decide what big ideas are most exciting to you and make them come alive for your students.
- ♦ Constantly focus on the simple ideas and "life lessons" that lead to great insights and invite students to apply those lessons in their daily lives.
- ♦ Meet non-science students where they truly are and move them from there.
- ♦ Talk to your colleagues around your institution. They will fully support such a change in requirements and philosophy.

Why to do it:
- ♦ You will change students' lives for the better.
- ♦ It's more fun and intellectually stimulating for faculty and students.
- ♦ It is not harder to teach, it's simply better.
- ♦ The administration will appreciate the Mathematics Department's contribution to the general education of students.
- ♦ Future alumni, donors, legislators, and citizens will see mathematics as an uplifting, positive, and important part of education and culture that deserves full support.

When to do it:
- ♦ *Now.*

Editor's note: The authors of this article have recently published the text *The Heart of Mathematics: An invitation to effective thinking* (Key Publishing, 2000), which is designed for students taking general education or teacher preparation courses in liberal arts mathematics or quantitative literacy. In a follow-up article they will discuss their experiences in choosing and communicating the deep mathematical ideas and processes that they advocate here.

The Boston College Mathematics Case Studies Project, *continued from page 11*

How can we train effective leaders of case study discussions?

How can we make the methodology more useful?

Elizabeth Brown, a mathematics Ph.D. student at Boston University who is working on the BCCase project, assisted in writing this article.

Call for Volunteers:
We seek a (small) number of individuals to use the present BCCase case studies with their graduate programs, and to give us careful feedback about how to improve the documents and methodology. We are particularly looking for faculty at research-oriented Ph.D. departments. We seek faculty who will help us address the points above, and we are open to them joining our project for the writing of additional cases.

To facilitate the process of feedback, we are asking interested faculty to attend a meeting, scheduled for June 25 (arrive June 24) to June 27 in Exeter, New Hampshire. We expect to be able to cover the room and board for faculty who are selected; individual departments must pay for all transportation and incidental expenses (a low-cost van from Logan airport to Exeter should be available). If you cannot attend the Exeter meeting, but you are still interested in the project, please let us know.

For more information, please contact:
David Foster, BCCase Project Administrator, bccase@bc.edu, or Solomon Friedberg, Project Director, friedber@bc.edu, or see our website http://www.bc.edu/bc_org/avp/cas/math/publicprojectPI/
We greatly value all comments and feedback. ■

The Boston College Mathematics Case Studies Project
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mathematics by the Harvard Mathematics Case Development Project, funded by the National Science Foundation.

Benefits of the Case Study Method:
The case study method of joint examination of concrete teaching scenarios offers a number of unique advantages. It gives graduate students a specific content in which to interpret overarching instructional beliefs. It is often the *implementation* of good ideas, rather than an absence of ideas, that makes teaching difficult; the case study gives graduate students a chance to think about how they would attempt do this. Effective teaching strategies vary widely from one proficient instructor to another; group discussions give students a chance to experience this first hand, to absorb new ideas, and to critically develop their own approach to the subject. The goal of the case study is for every participant to reflect on what good teaching is in the context of the given situation, not to arrive at unanimous consensus within the group.

An Example:
"Seeking Points," in which a graduate student is confronted by a student who received 5 out of 20 points on an exam for using the Power Rule to compute a derivative that the question stated should be solved using the *definition* of the derivative, is one of several case studies written for the project. Tested at Boston College (with a group of Boston-area graduate students), the University of Arizona, Dartmouth College, the University of Illinois at Chicago, Brown University, and Boston University, the case raises multiple issues including: why does one teach the difference quotient? how does one write a test question that really tests understanding of the difference quotient? how to handle disgruntled students? what is a fair division of partial credit? how to respond to correct prior knowledge, inappropriately used?

At Dartmouth, students and the discussion leader made a list of some 20 issues arising in the case. The issues were then prioritized, and a few were chosen to discuss in detail. The bulk of the discussion turned on the question of why the difference quotient should be taught, which the students did not feel was obvious. The Dartmouth students independently chose to return to the case study and its topics in a later meeting. The Brown discussion was similar, and the faculty T.A. training director chose to continue the discussion at a second meeting. At the University of Illinois at Chicago, discussion concerned the definition of "fair" grading, from both the student's and the TA's perspectives.
Some graduate student comments:

"In general, the suggestions were very relevant—this is a practical issue that arises often. It was beneficial to hear other TA's views on giving subjective credit to a student for showing understanding, and whether this conflicted with the need to grade within a test's boundaries." (UIC)

"Most [of the suggestions that surfaced in the group discussion] were relevant to my experiences. Members of the group tended to judge Sam [the undergraduate] harshly—'a grade is a grade'; he (she?) didn't do what the problem asked, etc. But we all became more uncertain as the group's leader began to question..." (UIC)

"One thing that challenged me is how to test a student's understanding of the math lesson." (UIC)

"On first reading, I thought Daniel [the TA] handled things pretty well, but discussion made me see more flaws." (Boston area grad students)

"My overall reaction is one of significant perplexity—this is a hard question, and I need to do some thinking about the philosophical issues involved and also what my concrete reaction to such a situation will be." (Brown University)

"Excellent case; it raised many important issues, but not in a blunt and obvious way—much like real-life situations." (Brown University)

Results to Date:
The trials of cases so far have shown that case studies can add a useful dimension to a math graduate student TA-training program, and that graduate students who have some teaching experience can successfully discuss complicated case scenarios. Also, many foreign students do well in such discussions.

The Next Step:
Some questions that the project would like to investigate are:

Does the case study method work with beginning teaching assistants, who may not have much perspective on teaching?

How can the use of the case studies be effectively combined with other TA training resources, such as a written guide to teaching?

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Notes on TA Training:

The Boston College Mathematics Case Studies Project

by Solomon Friedberg, Boston College

Introduction: Funded by a grant from FIPSE, the Fund for the Improvement of Post-Secondary Education, the Boston College Mathematics Case Studies (BCCase) Project is a three-year project to develop case studies for use in seminars in teacher training for mathematics graduate students. Like representative applications of a general theorem, these case studies provide concrete exercises students can use to develop and apply general principles of sound pedagogy. The members of the Development Team for the case studies are Solomon Friedberg, who is the Project Director, Avner Ash of The Ohio State University, Deborah Hughes Hallett of the University of Arizona, Margaret Kenney of Boston College, William McCallum of the University of Arizona, Jeremy Teitelbaum of the University of Illinois at Chicago, and Lee Zia of the University of New Hampshire.

Context: The growing demand for mathematically sophisticated college graduates has resulted in an increased demand for quality teaching of mathematics at the university level. Beginning with the calculus reform movement in the early nineties, improvement of mathematics teaching has become an issue of paramount importance, not only to mathematicians, but to the scientific community at large. Most graduate programs, including those in departments that are historically research oriented, now offer some kind of teacher training for their graduate students.

Such training is very much in the graduate students' best interest. Good teaching skills are important to the student's performance as a teaching assistant while in graduate school. They also help to develop communication and interpersonal skills which will be relevant to the graduate students' professional lives in academic or industrial spheres.

What is a Case Study?

A case study is a brief, realistic description of a situation that might occur in university mathematics teaching. Often created as an amalgam of several actual events, the case may describe one or more of the following: the instructor's behavior, students' behavior, student learning (or lack thereof), classroom (mis-) management, instructional content or practice, or the relationships between professor, undergraduate students, and graduate student(s). Each case raises a variety of pedagogic and communication issues to be explored through group discussion and analysis. Typically, this discussion is led by a moderator, who might provide the group with a starting point for discussion, help organize a listing of issues relevant to the case, etc.

The method of case studies has a long history of successful use in business and law. It has been used for 30 years in the training of university professors, particularly in the humanities, under a project headed by Roland Christensen of the Harvard Business School. Most recently, the method has been applied to the area of training for teachers of secondary

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