Some Solutions to HW 10

7.1.7

a) \(14! = 87178291200\) (the former answer is nicer since the reader can see where it came from).

c) \(11!4! = 958003200\) (there are 11 items: the 10 boys and the bunch of 4 girls which can be arranged in 4! ways).

d) \((11!−2!10!)4! = 783820800\) (all the arrangements allowed under (c) except those in which all the girls sit at one end or the other end).

7.2.12

a) \(\binom{8}{5} = 56\).

b) \(\binom{5}{3}\binom{3}{2} = 30\).

7.2.15

a) \(\binom{9}{6} = 84\).

b) \(\binom{9}{6} − \binom{7}{4} = 49\) (any group is OK, except a group which includes both exes (and 4 other people)).

(Alternatively: \(\binom{7}{6} + 2\binom{7}{5} = 49\) (choose from the 7 others, plus choose wife not husband and 5 others, plus choose husband not wife and 5 others.

Moral: there's more than one way to skin a counting problem.)

c) 0 spouse pairs there are 0 ways to choose 6 out of the remaining 3 people;

1 spouse pair 1 spouse pair equals 2 people, there are 0 ways to choose 4 out of the remaining 3 people;

2 spouse pairs 1 spouse pair equals 2 people, there are 0 ways to choose 4 out of the remaining 3 people;

2 spouse pairs 2 spouse pairs equals 4 people, there are \(\binom{3}{2}\) ways to pick the 2 spouse pairs and \(\binom{2}{2}\) ways to pick the remaining 2 people so we have \(\binom{3}{2}\binom{2}{2} = 9\) ways to do it this way;

3 spouse pairs 3 spouse pairs equals 6 people, there are \(\binom{3}{3}\) = 1 ways to choose the 3 spouse pairs and \(\binom{3}{0}\) = 1 ways to choose the remaining 0 people, so 1 \cdot 1 = 1 ways to do it this way.

So in total we have \(9 + 1 = 10\) ways to do this.