Some Solutions to HW 9

6.1.9

b) There are 25 people who bought grapefruit juice and orange juice; 35 people who bought grapefruit juice and apple juice; 33 people who bought grapefruit juice and citrus punch; 17 people who bought grapefruit juice, orange juice, and apple juice; 17 people who bought grapefruit juice, citrus punch, and apple juice; 1 person who bought all 4 (from Problem 6.1.9(a)). Therefore the answer is

\[ 100 - 25 - 35 - 33 + 17 + 2 + 9 - 1 = 34. \]

c) The sums of the number that bought two juices, minus the twice the sum of the number that bought three juices, plus three times the number that bought four juices, give the number that bought at least two juices

\[ 328 + 25 + 12 + 35 + 8 + 33 - 2(4 + 17 + 2 + 9) + 3 \cdot 1 = 380. \]

The sum of the numbers that bought three juices counts the number that bought four juices four times. Hence by subtracting three times the number that bought all four juices, we get the number that bought at least three juices

\[ 4 + 17 + 2 + 9 - 3 \cdot 1 = 29. \]

So the answer is 380 - 29 = 351.

d) We computed that in doing Problem 6.1.9(c): 29.

6.2.10

\[ 8 \times 8 \times 9 \times 10 \times 10 \times 10 \times 10 = 5,760,000. \]

6.3.11 Let \( a_i, 1 \leq i \leq 42 \) (recall that 42 = 6 \times 7), be the number of hours that Linda studies on day \( i \). Then

\[ 1 \leq a_1 < a_1 + a_2 < \cdots < a_1 + a_2 + \cdots + a_{42} \leq 50. \quad (1) \]

The only number from 1 to 50 which is divisible by 33 is 33 itself. Therefore either some number in (1) is divisible by (therefore equal to) 33, or all of the numbers are congruent to one of 1, \ldots, 32 modulo 33. Therefore (by the pigeon-hole principle) two of the numbers must be congruent to each other modulo 33. Suppose

\[ a_i \equiv a_j \pmod{33} \quad i < j. \]

Then \( a_{i+1} + \cdots + a_j \) is divisible by 33, which implies, since it is \( \leq 50 \), that \( a_{i+1} + \cdots + a_j = 33. \)