## Homework Set 2

1) Let $X$ and $Y$ be random variables on a discrete probability space $\Omega$ (here $\Omega$ is finite). Prove that $E(X+Y)=E(X)+E(Y)$.
2) Prove that $m(3)=7$ using the sketch from class (recall that $m(3)$ is the minimum number of edges in a 3 -uniform hypergraph that is not 2-colorable).
3) Show that the deterministic algorithm to construct signs for unit vectors so that the resulting vector has small norm works.
4) (somewhat hard) Prove that there is a positive constant $c$ so that every set $A$ of $n$ nonzero reals contains a subset $B \subset A$ of size $|B| \geq c n$ so that there are no $b_{1}, b_{2}, b_{3}, b_{4} \in B$ satisfying

$$
b_{1}+2 b_{2}=2 b_{3}+2 b_{4}
$$

