Homework Set 2

1) Let X and Y be random variables on a discrete probability space Ω (here Ω is finite). Prove that E(X + Y) = E(X) + E(Y).

2) Prove that m(3) = 7 using the sketch from class (recall that m(3) is the minimum number of edges in a 3-uniform hypergraph that is not 2-colorable).

3) Show that the deterministic algorithm to construct signs for unit vectors so that the resulting vector has small norm works.

4) (somewhat hard) Prove that there is a positive constant c so that every set A of n nonzero **reals** contains a subset $B \subset A$ of size $|B| \ge cn$ so that there are no $b_1, b_2, b_3, b_4 \in B$ satisfying

$$b_1 + 2b_2 = 2b_3 + 2b_4.$$