Homework Set 5

1) Prove that $G(n,p)$ has minimum degree at most one whp if

$$p(n) = \frac{\ln n + \ln\ln n - w(n)}{n}$$

and has minimum degree at least two whp if $p(n) = (\ln n + \ln\ln n + w(n))/n$. Here $w(n) \to \infty$ as $n \to \infty$.

2) Let H be a graph, and let $n > |V(H)|$ be an integer. Suppose that there is a graph on n vertices and t edges containing no copy of H, and suppose that $tk > n^2 \log_e n$. Show that there is a coloring of the edges of the complete graph on n vertices by k colors with no monochromatic copy of H.

3) Let $e, f_1, \dots f_t$ be edges in a k-uniform hypergraph $H = (V, E)$ such that $e \cap f_i \neq \emptyset$ for all i. Suppose that V is randomly and independently two colored with red and blue. Let R_e be the event that e is red and R_{f_i} be the event that f_i is red. Prove that

$$P(R_e \mid \bigwedge_i \overline{R}_{f_i}) \leq P(R_e).$$

4) Let $G = (V, E)$ be a simple graph and suppose each $v \in V$ is associated with a set $S(v)$ of colors of size at least $10d$, where $d \geq 1$. Suppose, in addition, that for each $v \in V$ and $c \in S(v)$ there are at most d neighbors u of v such that $c \in S(v)$. Prove that there is a proper coloring of G assigning to each vertex v a color from its class $S(v)$.