## Homework Set 6

1) Let $G$ be a graph and let $P$ denote the probability that a random subgraph of $G$ obtained by picking each edge of $G$ with probability $1 / 2$, independently, is connected (and spanning). Let $Q$ denote the probability that in a random two-coloring of $G$, where each edge is chosen, randomly and independently, to be either red or blue, the red graph and the blue graph are both connected (and spanning). Is $Q \leq P^{2}$ ?
2) Let $F_{1}, \ldots, F_{k}$ be $k$ intersecting families of subsets of $[n]$. Prove, using Kleitman's inequalities, that

$$
\left|\cup_{i=1}^{k} F_{i}\right| \leq 2^{n}-2^{n-k}
$$

Show an (easy) example that achieves the bound.
Hint: use induction on $k$; you may also assume that each $F_{i}$ is maximal, and hence monotone increasing (why?).
3) Show that the probability that in the random graph $G(2 k, 1 / 2)$ the maximum degree is at most $k-1$ is at least $1 / 4^{k}$.
4) Suppose that $\Omega=A \cup B$ is a probability space, where $A \cap B=\emptyset$. Suppose that $f: \Omega \rightarrow R$ is a random variable. Let $a$ be the expected value of $f$ on $A$, $b$ be the expected value of $f$ on $B$, and $c$ be the expected value of $f$ on $A \cup B$ (so $a=\left(\sum_{x \in A} f(x) P(x)\right) / P(A)$ etc.) Define $Z(x)$ to be $a$ if $x \in A$ and $b$ if $x \in B$. Prove that $E(Z)=c$. Conclude that the edge exposure martingale $X_{0}, \ldots, X_{m}$ for some graph function $f$ satisfies $E\left(X_{i} \mid X_{i-1}, \ldots, X_{0}\right)=X_{i-1}$.

