**Homework Set 7**

1) Let $d \geq 3$ be a constant and $\rho = p(n-1)$. Prove the following: If $\rho \ll n^{-1/d}$ then $G(n,p)$ does not have a vertex of degree $d$ almost surely and if $\rho \gg n^{-1/d}$ then $G(n,p)$ has a vertex of degree $d$ almost surely.

2) Let $p$ be a prime congruent to 1 mod 4 and $G_p$ the graph with vertex set $GF(p)$ and $ij$ forming an edge iff $i-j$ is a quadratic residue mod $p$. Show that $G_p$ is well-defined and is regular of degree $(p-1)/2$. Let $B$ and $C$ be disjoint sets of vertices in $G_p$. Prove that

$$\left| e(B,C) - \frac{1}{2}|B||C| \right| \leq \frac{1}{2}|B|^{1/2}|C|^{1/2}p^{1/2}.$$

3) Let $G=(V,E)$ be an $(n,d,\lambda)$-graph and $k|n$. Suppose that $c$ is a $k$-coloring of $V$ so that each color appears precisely $n/k$ times. Prove that there is a vertex of $G$ which has a neighbor of each of the $k$ colors, provided $k\lambda \leq d$.

4) Let $\mathcal{F}$ be a family of graphs on vertex set $[2t]$ and suppose that for every two graphs in $\mathcal{F}$ there is a perfect matching in their intersection (of their edge sets). Prove that

$$|\mathcal{F}| \leq 2^{\binom{2t}{2}-t}.$$