**Homework 2**

1) Fix integers $2 \leq s \leq t$. Prove that there is a constant $c = c(s,t)$ such that $\mathrm{ex}(n, K_{s,t}) < cn^{2-1/t}$.

2) Let $S = [mn]$ and suppose that $S$ is partitioned into $m$ sets $A_1, \dots, A_m$ each of size $n$. Suppose that $B_1, \dots, B_m$ is another partition of $[mn]$ into sets of size $n$. Show that the sets $A_i$ can be renumbered in such a way that $A_i \cap B_i \neq \emptyset$.

3) Let $\mathcal{A} \subset 2^{[n]}$ be an intersecting sperner family such that $A \cup B \neq [n]$ for every $A, B \in \mathcal{A}$. Prove that

$$|\mathcal{A}| \leq \binom{n-1}{\lfloor n/2 \rfloor - 1}.$$

4) Let $x_1, \dots, x_n$ be a collection of reals numbers each at least 1. For $A \subset [n]$, let $x_A = \sum_{i \in A} x_i$. What is the maximum number of such sums that pairwise differ by less than one.

5) Let $n^2 + 1$ points be given in $R^2$. Prove that among them there is a sequence of $n+1$ points $(x_1, y_1), \dots, (x_{n+1}, y_{n+1})$ for which $x_1 \leq \cdots \leq x_{n+1}$ and $y_1 \geq \cdots \geq y_{n+1}$, or a sequence of $n+1$ points for which $x_1 \leq \cdots \leq x_{n+1}$ and $y_1 \leq \cdots \leq y_{n+1}$.

6) Let $k \geq 1$ be an integer and $G = (A, B, E)$ be a bipartite graph with parts $A, B$. Suppose that for every $S \subset A$ we have $|N(S)| \geq k|S|$. Prove that there is a collection of $|A|$ pairwise vertex disjoint stars, each with $k$ edges and centers in $A$.