Homework 3

1) Let $[S, T]$ and $[S', T']$ be source/sink cuts in a network.
a) Prove that $c(S \cup S', T \cap T') + c(S \cap S', T \cup T') \leq c(S, T) + c(S', T')$.
b) Suppose that $[S, T]$ and $[S', T']$ are minimum cuts. Show that no edge between $S - S'$ and $S' - S$ has positive capacity.

2) Find a circular ternary sequence of length 27 so that each possible ternary ordered triple occurs as three consecutive positions of the sequence.

3) Determine $N(C_{2n})$, the minimum length of vectors with entries from $\{0, 1, *\}$ that provide an addressing for C_{2n}.

4) Determine $f(x) = \sum_{n=1}^{x} \mu(n)\lfloor x/n \rfloor$ for all positive integers x.

5) Let a and b be integers with $2a \leq b$ and b even. Suppose that χ is a proper edge-coloring of K_a with $b - 1$ colors. Prove that there is a proper edge-coloring χ' of K_b with $b-1$ colors such that the restriction of χ' to some subgraph of K_b isomorphic to K_a is identical to χ.