Homework Set 3

1) For integers $1 < k < n$, prove that $(n/k)^k < \binom{n}{k} < (ne/k)^k$ and $n! > (n/e)^n$.

2) Prove that there is an absolute constant $c > 0$ such that: Every n by n matrix A with pairwise distinct entries has a permutation of its rows so that no column of the permuted matrix contains an increasing subsequence of length at least $c\sqrt{n}$.
Hint: Prove that the probability that a fixed column, after permuting, has a long increasing subsequence is less than $1/n$. Use the estimates in Problem 1.

3) Prove, via the alteration method, that $R(k,k) > (1+o(1))(k/e)2^{k/2}$ and $R(4,k) > \Omega((k/\ln k)^2)$. (this improves the bound in a previous homework)

4) Prove that every three uniform hypergraph with n vertices and $m \geq n/3$ edges contains an independent set of size at least $(2/\sqrt{m})(n/3)^{3/2}$.