Homework Set 1

1) Prove that $r(3; q) < q!e + 1$.

2) Prove or disprove: In every 3-coloring of the points of the plane, there are two points at distance 1 with the same color.

3) Prove or disprove: In every 2-coloring of the points of the plane, there is a monochromatic regular triangle with side 1.

4) Let $k, r \geq 1$ be given. Prove that there is an $N(k, r)$ such that if $n \geq N(k, r)$ and $[n]$ is k-colored, then we can find natural numbers $a, d_1, \ldots, d_r$ such that all sums

$$a + d_{i_1} + \cdots d_{i_\nu} \quad (1 \leq i_1 < \cdots < i_\nu \leq r.\ 0 \leq \nu \leq r)$$

have the same color (and, of course, $a + d_1 + \cdots + d_r \leq n$).

5) Prove Schur's theorem from class with x, y, z distinct.