**Homework Set 2**

1) Prove the Hales Jewett theorem for combinatorial $m$-spaces. Hint: use induction on $m$. You may assume the Hales Jewett Theorem, i.e. the case $m = 1$. For the induction step from $m$ to $m + 1$, consider words of length $M + N$, where $M = HJ(1, r, t)$ and $N = HJ(m, r^{t^M}, t)$.

2) Let $q$ be a prime power and $r \geq 1$, $k \geq 0$. Prove that if $n$ is sufficiently large, then the following holds: Every $r$-coloring of the points of the affine $n$-dimensional space over $GF(q)$ yields a monochromatic $k$-dimensional subspace.

3) Let $k, r \geq 1$. Construct, either using Ramsey's theorem or using 2) above, a $k$-uniform hypergraph with chromatic number at least $k + 1$ and in which every two edges have at most one point in common. Recall that the chromatic number is the least number of vertex classes that parttion the vertex set, where each class is an independent set.

4) Using induction on $s$, prove the following modular version of the sunflower lemma: Let $\mathcal{F}$ be a family of $s$-sets of an $n$-set, and suppose that $|\mathcal{F}| > F(n, k, s)$, where

$$F(n, k, s) = (nk)^{s/2} \cdot \frac{1 \cdot 3 \cdots (s-1)}{2 \cdot 4 \cdots s}$$

if $s$ is even and

$$F(n, k, s) = n^{(s-1)/2} k^{(s+1)/2} \cdot \frac{1 \cdot 3 \cdots s}{2 \cdot 4 \cdots (s-1)}$$

if $s$ is odd. Then $\mathcal{F}$ contains a sunflower with $k + 1$ petals and a core $Y$ such that $|Y| \equiv s + 1 (mod\, 2)$.