Homework Set 3

1) Suppose that $\mathcal{F}$ is an intersecting family of k-sets of $[n]$, and $S_{i,j}(\mathcal{F})$ is a star of size $\binom{n-1}{k-1}$. Prove that $\mathcal{F}$ is also a star. This implies the characterization of equality in the original proof of the Erdős-Ko-Rado theorem.

2) Prove the characterization of equality in the Erdős-Ko-Rado theorem using Katona's permutation method. First you must prove (easy) that for equality to hold within a permutation, all sets contains a fixed point. Then you must prove (harder) that this is the same point for different permutations.

3) (i) Prove that every positive integer m has a unique k-cascade representation

$$m = \binom{a_k}{k} + \cdots + \binom{a_t}{t}$$

where $a_k > a_{k-1} > \cdots > a_t \geq t \geq 1$.

(ii) Prove that the shadow of the first m elements in the colex order on k-sets is the first m' elements of the colex order on $k-1$ sets, where $m' = \binom{a_k}{k-1} + \cdots + \binom{a_t}{t-1}$ and $\binom{a}{b} = 0$ if $a \leq b$.

4) Let $\mathcal{F}$ be a family of k-sets. Prove that the shadow of $S_{i,j}\mathcal{F}$ is contained in $S_{i,j}\partial\mathcal{F}$.