## Homework Set 4

1) Let $k<n$ be positive. Prove that $(n / k)^{k}<\binom{n}{k}<(n e / k)^{k}$.
2) Suppose that there are $m$ red clubs $R_{1}, \ldots, R_{m}$, and $m$ blue clubs $B_{1}, \ldots, B_{m}$ in a town of $n$ citizens. Assume that these clubs satisfy the following rules:
(i) $\left|R_{i} \cap B_{i}\right|$ is odd for every $i$
(ii) $\left|R_{i} \cap B_{j}\right|$ is even for every $i \neq j$.

Prove that $m \leq n$. Also find an example where $m=n$.
3) Let $A$ be a $2 n \times 2 n$ matrix with zeros in the diagonal and $\pm 1$ everywhere else. Prove that $A$ is nonsingular (i.e. invertible) over the reals.
4) Construct a 2-distance set $S \subseteq \mathbf{R}^{n}$ of size $n(n+1) / 2$. What are the two distances? Generalize this construction to obtain a large $s$-distance set in $\mathbf{R}^{n}$.
5) Give an elementary explicit construction (not using any results from class) showing that the Ramsey number $R(t, t)>(t-1)^{2}$.

