## Homework Set 4

1) Let k < n be positive. Prove that  $(n/k)^k < \binom{n}{k} < (ne/k)^k$ .

2) Suppose that there are m red clubs  $R_1, \ldots, R_m$ , and m blue clubs  $B_1, \ldots, B_m$  in a town of n citizens. Assume that these clubs satisfy the following rules:

(i)  $|R_i \cap B_i|$  is odd for every *i* 

(ii)  $|R_i \cap B_j|$  is even for every  $i \neq j$ .

Prove that  $m \leq n$ . Also find an example where m = n.

3) Let A be a  $2n \times 2n$  matrix with zeros in the diagonal and  $\pm 1$  everywhere else. Prove that A is nonsingular (i.e. invertible) over the reals.

4) Construct a 2-distance set  $S \subseteq \mathbf{R}^n$  of size n(n+1)/2. What are the two distances? Generalize this construction to obtain a large *s*-distance set in  $\mathbf{R}^n$ .

5) Give an elementary explicit construction (not using any results from class) showing that the Ramsey number  $R(t,t) > (t-1)^2$ .