Homework Set 7

1) Let G be an n-vertex graph that contains no triangle. Prove that for every edge uv of G, $d(u) + d(v) \leq n$. Sum this inequality over all edges to obtain another proof of Turáns theorem (for just K_3) that $\mathrm{ex}(n, K_3) = \lfloor n^2/4 \rfloor$ for $n \geq 3$. Hint: you will need to use the Cauchy-Schwarz inequality, or convexity of binomial coefficients.

2) Suppose that (A, B) is an ϵ-regular pair with density d. Suppose $A' \subset A$ and $B' \subset B$ such that $|A'| = |A|/2$ and $|B'| = |B|/2$. Is there a $\delta > 0$ such that (A', B') is a δ-regular pair? If you answer yes, then give an explicit formula for δ.

3) Let P_4 be the path with four vertices and three edges. Determine $\mathrm{ex}(n, P_4)$ exactly for all n.

4) The wheel W_k is the graph obtained from the cycle C_k by adding a new vertex adjacent to all vertices of the cycle. Determine the asymptotics for $\mathrm{ex}(n, W_k)$ for all $k \geq 3$.