Homework Set 1

1) Prove that if there is a real p, $0 \le p \le 1$ such that

$$\binom{n}{k} p^{\binom{k}{2}} + \binom{n}{t} (1-p)^{\binom{t}{2}} < 1,$$

then $R(k,t) > n$. Conclude that

$$R(4,t) > c\, t^{3/2}/(\log t)^{3/2}$$

for an absolute constant c.

2) Suppose $n \ge 4$ and H is an n-uniform hypergraph with at most $4^{n-1}/3^n$ edges. Prove that there is a coloring of the vertices of H by four colors so that all four colors appear in every edge.

3) Let $\{(A_i, B_i), 1 \le i \le h\}$ be a family of pairs of subsets of the set of integers such that $|A_i| = k$ for all i and $|B_i| = l$ fr all i, $A_i \cap B_i = \emptyset$ and $(A_i \cap B_j) \cup (A_j \cap B_i) \ne \emptyset$ for all $i \ne j$. Prove that

$$h \le \frac{(k+l)^{k+l}}{k^k l^l}.$$

4) (Hard) Show that there is an absolute constant c so that for each $k > 1$, every tournament that satisfies S_k has at least $c\, k 2^k$ vertices. Hint: Show that for every set W of at most $k-1$ vertices, there are at least $k+1$ vertices that all point to each vertex of W.