

## Homework Set 2

- 1) Let  $X$  and  $Y$  be random variables on a discrete probability space  $\Omega$ . Prove that  $E(X + Y) = E(X) + E(Y)$ .
- 2) Show that the deterministic algorithm to construct signs for unit vectors so that the resulting vector has small norm works.
- 3) Suppose  $n \geq 2$  and let  $H = (V, E)$  be an  $n$ -uniform hypergraph with  $|E| = 4^{n-1}$  edges. Show that there is a coloring of  $V$  by four colors so that no edge is monochromatic.
- 4) Prove that there is a positive constant  $c$  so that every set  $A$  of  $n$  nonzero reals contains a subset  $B \subset A$  of size  $|B| \geq cn$  so that there are no  $b_1, b_2, b_3, b_4 \in B$  satisfying

$$b_1 + 2b_2 = 2b_3 + 2b_4.$$