Homework 2

1) Prove that a graph with n vertices and e edges has at least $\frac{e}{3n}(4e - n^2)$ triangles.

2) Let $S = [mn]$ and suppose that S is partitioned into m sets $A_1, \dots, A_m$ each of size n. Suppose that $B_1, \dots, B_m$ is another partition of $[mn]$ into sets of size n. Show that the sets A_i can be renumbered in such a way that $A_i \cap B_i \neq \emptyset$.

3) Let $\mathcal{A} \subset 2^{[n]}$ be an intersecting sperner family such that $A \cup B \neq [n]$ for every $A, B \in \mathcal{A}$. Prove that

$$|\mathcal{A}| \leq \binom{n-1}{\lfloor n/2 \rfloor - 1}.$$

4) Let $x_1, \dots, x_n$ be a collection of reals numbers each at least 1. For $A \subset [n]$, let $x_A = \sum_{i \in A} x_i$. What is the maximum number of such sums that pairwise differ by less than one.

5) Let $n^2 + 1$ points be given in R^2. Prove that there is a sequence of $n + 1$ points $(x_1, y_1), \dots, (x_{n+1}, y_{n+1})$ for which $x_1 \leq \dots \leq x_{n+1}$ and $y_1 \geq \dots \geq y_{n+1}$, or a sequence of $n + 1$ points for which $x_1 \leq \dots \leq x_{n+1}$ and $y_1 \leq \dots \leq y_{n+1}$.