Homework 3

1) Let $[S, T]$ and $[S', T']$ be source/sink cuts in a network.
a) Prove that $c(S \cup S', T \cap T') + c(S \cap S', T \cup T') \leq c(S, T) + c(S', T')$.
b) Suppose that $[S, T]$ and $[S', T']$ are minimum cuts. Show that no edge between $S - S'$ and $S' - S$ has positive capacity.

2) Find a circular ternary sequence of length 27 so that each possible ternary ordered triple occurs as three consecutive positions of the sequence.

3) Determine $N(C_{2n})$, the minimum length of vectors with entries from $\{0, 1, *\}$ that provide an addressing for C_{2n}.

4) Determine $f(x) = \sum_{n=1}^{x} \mu(n) \lfloor x/n \rfloor$ for all positive integers x.

5) Let $M(n, k)$ be the maximum possible value of the permanent of an n by n 0-1 matrix with exactly k 1's in each row and column.
a) Prove that $M(n, k) \geq k!$.
b) Prove that $M(k) = \lim_{n \to \infty} M(n, k)^{1/n} = (k!)^{1/k}$.