Homework 4

1) Suppose that f is a function from $\mathbf{N}$ to $\mathbf{N}$ such that $f(a+b) \geq f(a)+f(b)$. Prove that $\lim_{n\to\infty} f(n)/n$ exists (possibly ∞).

2) Problem 11D

3) Problem 11E

4) Prove that for all odd $k > 1$, there exists $r_0 = r_0(k)$ such that for every even $r > r_0$, every r-regular graph contains a k-regular subgraph. You may assume that the prime numbers occur frequently.

5) Prove that the Bell number satisfies the equation

$$B(n) = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}.$$