Homework 6

1) Prove that there is a constant $c > 0$ such that every graph on n vertices and at least $cn^{3/2}$ edges contains a subset S of 10 vertices such that every pair of vertices in S has at least 10 common neighbors. Hint: Dependent Random Choice

2) Prove Ruzsa's Lemma: For every three sets U, V, W we have

$$|U||V - W| \leq |U + V||U + W|.$$

3) Solve Problem 38A (page 540) using the proof of Baranyai's theorem presented in class.

4) Prove the $(6, 3)$ theorem from the induced matching lemma. I.e. prove that for every $\epsilon > 0$ there exists n_0 such that for $n > n_0$, every 3-uniform hypergraph H on n vertices in which every six points span at most three edges satisfies $|E(H)| < \epsilon n^2$. You may use the induced matching lemma: that in an n by n bipartite graph comprising n induced matchings, the total number of edges is at most γn^2 (here $\gamma > 0$ is fixed and n is large).