MATH 310, Exam 1 – Version B Name:

NO CALCULATORS. For full credit, SHOW ALL WORK.

1. (10 points) Solve the following linear system.

$$x_1 - 2x_2 + 5x_3 = 3$$
$$-3x_1 + 6x_2 - 14x_3 = -7$$

2. (10 points) Consider the matrix A, with its reduced row echelon form U, given below.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ -5 & 12 & -7 \\ 3 & -5 & 13 \end{bmatrix} \xrightarrow{\text{RREF}} U = \begin{bmatrix} 1 & 0 & 11 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Consider the linear transformation L_A defined by $L_A(\mathbf{x}) := A\mathbf{x}$.

(a) For which values of n and m do we have $L_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m$?

(b) Is L_A one-to-one? Justify your answer.

(c) Is L_A onto? Justify your answer.

(d) Is $\left\{ \begin{bmatrix} 1 \\ -2 \\ -5 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 12 \\ -5 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -7 \\ 13 \end{bmatrix} \right\}$ linearly dependent or linearly independent? Explain.

3. (10 points) Consider the matrix B given by

$$B = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & -5 \\ 2 & 6 & 11 \end{bmatrix}.$$

(a) Compute the LU factorization of B.

(b) Find the inverse of B.

4. (10 points) Let A be a 3×3 matrix satisfying

$$A^{-1} = \begin{bmatrix} -1 & 5 & 1 \\ 0 & 2 & 1 \\ 1 & -2 & 0 \end{bmatrix}, \quad \text{and let} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Solve the matrix equation $A\mathbf{x} = \mathbf{b}$.

(b) Solve the matrix equation $A^2\mathbf{x} = \mathbf{b}$.

5. (10 points) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear transformation which first reflects points through the horizontal x_1-axis and then rotates points about the origin through an angle of $\pi/4$ radians (i.e. 45 degrees) in the counterclockwise direction.

(a) Sketch $T(\mathbf{e}_1)$, where $\mathbf{e}_1 := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(b) Find the standard matrix of T.