MATH 310, Exam 2 – Version A   Name:

NO CALCULATORS. For full credit, SHOW ALL WORK.

1. (10 points) Consider the matrix C, with its reduced row echelon form U, given below.

$$C = \begin{bmatrix} 2 & 3 & 6 & -4 & 3 & 4 \\ 3 & 3 & 9 & -3 & 0 & 3 \\ -5 & -3 & -15 & 1 & 6 & -1 \\ 3 & 3 & 9 & -3 & -3 & 0 \end{bmatrix} \xrightarrow{\text{RREF}} U = \begin{bmatrix} 1 & 0 & 3 & 1 & 0 & 2 \\ 0 & 1 & 0 & -2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the nullspace of C.

(b) Find a basis for the column space of C.

(c) Find a basis for the row space of C.

(d) Find the dimension of the image of the linear transformation $L_C : \mathbb{R}^6 \to \mathbb{R}^4$ given by $L_C(\mathbf{x}) := C\mathbf{x}$.

2. (10 points) (a) Find the determinant of A, where

$$A := \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 3 & 7 \\ 0 & -2 & 0 & 1 \\ 3 & 1 & 0 & 0 \end{bmatrix}.$$

(b) A student is given a 3×3 matrix B and performs on B the row operations $\mathbf{r_1} \leftrightarrow \mathbf{r_2}$ (i.e. swaps rows 1 and 2), $\mathbf{r_3} \mapsto \mathbf{r_3} + (-2)\mathbf{r_1}$ (i.e. replaces row 3 with itself plus -2 times row 1) and $\mathbf{r_2} \mapsto 3\mathbf{r_2}$ (i.e. replaces row 2 with 3 times itself). This sequence of row operations transforms B into the following matrix:

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 4 & 7 \end{bmatrix}.$$

What is the determinant of the original matrix B?

3. (10 points) Let $\mathcal{B} := \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$ and $\mathcal{B}' := \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix} \right\}$, which are each bases of $\mathbb{R}^2$.

(a) Find $\mathbf{x} \in \mathbb{R}^2$, if $[\mathbf{x}]_{\mathcal{B}'} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(b) Find $[\mathbf{y}]_{\mathcal{B}}$, if $\mathbf{y} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$.

(c) Find the change of basis matrix $\underset{\mathcal{B} \leftarrow \mathcal{B}'}{P}$.

(There are more questions on the back of this page)

4. (10 points) (a) Find all the eigenvalues of the matrix $D := \begin{bmatrix} -4 & 2 \\ 3 & 1 \end{bmatrix}$.

(b) Given that $\lambda = 6$ is an eigenvalue of $E := \begin{bmatrix} 3 & 0 & -6 \\ 0 & 4 & -2 \\ 1 & -6 & 2 \end{bmatrix}$, find one corresponding eigenvector.

(c) Is $\mathbf{u} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}$ an eigenvector of the matrix E from part (b)? If so, what is the corresponding eigenvalue?

(d) Compute $E^{10} \cdot \mathbf{u}$, where $\mathbf{u} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}$ and E is the matrix from part (b).

5. (10 points) Suppose that λ and μ are real numbers for which the matrix

$$A = \begin{bmatrix} \lambda & 1 & 2 \\ \mu & 9 & 5 \\ 6 & 7 & 4 \end{bmatrix}$$

satisfies $\det A = -5$. Use Cramer's rule to find x_1, where

$$\begin{bmatrix} \lambda & 1 & 2 \\ \mu & 9 & 5 \\ 6 & 7 & 4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$