1. (20 points) Solve the initial value problem:
\[ \frac{dy}{dx} = \frac{y^2}{x^2}, \quad y(1) = -1. \]

2. (20 points) Find the general solution of:
\[ \cos(\theta) \frac{dr}{d\theta} + \sin(\theta)r = \cos^3(\theta). \]

3. (20 points) Consider the initial value problem:
\[ \frac{dy}{dx} = \frac{y^2}{x^2}, \quad y(1) = -1. \]

(a) (8 points) With a step size of \( h = 1 \) use Euler’s method to approximate the solution at \( x = 2 \).

(b) (10 points) With a step size of \( h = 1/2 \) use Euler’s method to approximate the solution at \( x = 2 \).

(c) (2 points) Let \( E(h) \) denote the error in Euler’s method with step size \( h \) in approximating solutions at \( x = 2 \). Roughly, what is the ratio \( \frac{E(h/2)}{E(h)} \)? Why?

4. (20 points) Consider the second order ODE:
\[ y'' - 6y' + 9y = 0. \]

(a) (10 points) Find two linearly independent solutions of this ODE. Be sure to prove that they are linearly independent.

(b) (2 points) Write down the general solution of this ODE.

(c) (8 points) Write down the unique solution of this ODE subject to the initial conditions:
\[ y(0) = 4, \quad y'(0) = 11. \]

5. (20 points) A parachutist whose mass is 100 kg drops from a hovering helicopter and falls toward the ground under the influence of gravity. At 1000 m above the ground her velocity is 9.8 m/s and she opens her chute. Assume that the force due to air resistance is proportional to the velocity of the parachutist with the proportionality constant \( b = 50 \text{ N-sec/m} \) when the chute is open. Find an equation for the time (after the chute has opened) when she reaches the ground. What is an approximate value of this time?