MATHEMATICS 586: Homework 1
University of Illinois at Chicago (Professor Nicholls)
Spring 2024

Due Friday, February 2 by 2pm.

1. (Higham Exercise 4.4) In the case where $f(x)$ is the density for the exponential distribution with parameter $\lambda = 1$, show that the quantile $z(p)$,
$$\int_{-\infty}^{z(p)} f(x)\, dx = p,$$
satisfies $z(p) = -\log(1-p)$.

2. (Higham Exercise 6.4) Verify
$$E\left[\mu\delta t + \sigma\sqrt{\delta t}Y_i - \frac{1}{2}\sigma^2\delta t Y_i^2\right] = \mu\delta t - \frac{1}{2}\sigma^2\delta t$$
and
$$\text{var}\left(\mu\delta t + \sigma\sqrt{\delta t}Y_i - \frac{1}{2}\sigma^2\delta t Y_i^2\right) = \sigma^2\delta t + \text{higher powers of } \delta t.$$

3. (Higham Exercise 8.3) Confirm that
$$C(S,t) = SN(d_1) - Ee^{-r(T-t)}N(d_2)$$
satisfies:
   (a) $C(S,t) = \max\{S(T) - E, 0\}$ (Hint: Take the limit $t \to T^-$)
   (b) $C(0,t) = 0, \quad 0 \leq t \leq T$ (Hint: Take the limit $S \to 0^+$)
   (c) $C(S,t) \approx S, \quad S \gg 1$ (Hint: Take the limit $S \to \infty$)

4. (Higham Programming Exercise 8.1) Use `ch08.m` to produce graphs illustrating the limits
$$\lim_{t\to T^-} C(S,t) = \max\{S(T) - E, 0\}$$
and
$$\lim_{S\to\infty} C(S,t) = S$$
as established in Exercise 8.3.

   Currently, the code can be downloaded from:

   `http://personal.strath.ac.uk/d.j.higham/option_book.html`

5. Wilmott, Howison, & Dewynne, Chapter 2, # 1.

6. Wilmott, Howison, & Dewynne, Chapter 3, # 2 (a)–(d).

7. Wilmott, Howison, & Dewynne, Chapter 3, # 3.