Quiz 11 MATH 210, CALCULUS III, SUMMER 2015

NAME:

Problem 1. Find the AVERAGE VALUE of

$$f(x,y,z) = 3x + 2y - z$$

over the solid $D = \{(x,y,z) : 0 \le x \le 2, 1 \le y \le 2, 0 \le z \le 1\}$.
Hint: Average value is not just the integral.

Volume of D: $(2-0)(2-1)(1-0) = 2\cdot 1\cdot 1 = 2$

Average: $\frac{1}{2}\int_0^2\int_1^2\int_0^1 3x+2y-z\,dz\,dy\,dx = \frac{1}{2}\int_0^2\int_1^2 (3x+2y)z - \frac{z^2}{2}\Big|_0^1 dy\,dx$

$= \frac{1}{2}\int_0^2\int_1^2 (3x+2y)\cdot 1 - \frac{1^2}{2} - \left((3x+2y)(0) - \frac{0^2}{2}\right)dy\,dx = \frac{1}{2}\int_0^2\int_1^2 3x+2y-\frac{1}{2}\,dy\,dx$

$= \frac{1}{2}\int_0^2 (3x-\frac{1}{2})y + y^2\Big|_1^2 dx = \frac{1}{2}\int_0^2 (3x-\frac{1}{2})\cdot 2 + 2^2 - \left((3x-\frac{1}{2})\cdot 1 + 1^2\right)dx$

$= \frac{1}{2}\int_0^2 6x - 1 + 4 - 3x + \frac{1}{2} - 1\,dx = \frac{1}{2}\int_0^2 3x + \frac{5}{2}\,dx = \frac{1}{2}\left[\frac{3}{2}x^2 + \frac{5}{2}x\right]_0^2$

$= \frac{1}{2}\left(\frac{3}{2}(2)^2 + \frac{5}{2}(2)\right) - \frac{1}{2}\left(\frac{3}{2}(0)^2 + \frac{5}{2}(0)\right) = \frac{1}{2}(6+5) = \frac{11}{2}$

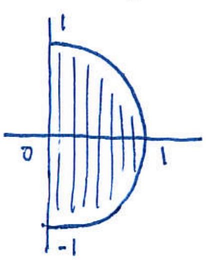
Problem 2. Rewrite the integral

$$\int_0^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^1 \sqrt{x^2+y^2}\,dz\,dy\,dx$$

in cylindrical coordinates.
Extra credit: (1 extra point) Solve the integral.

$R = \{(x,y,z) : 0 \le x \le 1, -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}, 0 \le z \le 1\}$

$= \{(r,\theta,z) : 0 \le r \le 1, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, 0 \le z \le 1\}$



$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^1\int_0^1 \sqrt{r^2}\,dz\,r\,dr\,d\theta$

Extra Credit: $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^1\int_0^1 r^2\,dz\,dr\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^1 r^2 z\Big|_0^1 dr\,d\theta$

$= \int_{-\pi/2}^{\pi/2}\int_0^1 r^2(1) - r^2(0)\,dr\,d\theta = \int_{-\pi/2}^{\pi/2}\int_0^1 r^2\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{r^3}{3}\Big|_0^1 d\theta$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1^3}{3} - \frac{0^3}{3}\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{3}\,d\theta = \frac{1}{3}\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{3}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \frac{1}{3}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \frac{\pi}{3}$