

Quiz 12

MATH 210, CALCULUS III, SUMMER 2015

NAME:

Problem 1. Solve the relation for x and y and compute the Jacobian:

$$u = x + y$$

$$v = 2x - y$$

$$x = u - y \quad y = 2x - v$$

$$x = u - (2x - v) \quad y = 2\left(\frac{u+v}{3}\right) - v$$

$$x = u + v - 2x \quad y = \frac{2u + 2v}{3} - \frac{3v}{3}$$

$$3x = u + v \quad y = \frac{2u - v}{3}$$

$$x = \frac{u+v}{3}$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{vmatrix} = \frac{1}{3}\left(-\frac{1}{3}\right) - \frac{1}{3}\left(\frac{2}{3}\right)$$

$$= -\frac{1}{9} - \frac{2}{9}$$

$$= -\frac{3}{9} = \boxed{-\frac{1}{3}}$$

Problem 2. Find the gradient vector field $F = \nabla\phi$ for $\phi = \frac{x^2+2}{2}$ and sketch it.

$$\nabla\phi = \left\langle \frac{2x}{2}, 0 \right\rangle = \langle x, 0 \rangle$$

