

Quiz 14

MATH 210, CALCULUS III, SUMMER 2015

NAME:

Problem 1. Use Green's Theorem to find the circulation of $\mathbf{F} = \langle -2y, x \rangle$ on the circle $x^2 + y^2 = 1$.

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} dA = \iint_R 1 - (-2) dA = 3 \iint_R dA$$

$$\iint_R dA = \text{area of } R = \text{area of the circle with radius 1} \\ = \pi(1)^2 = \pi$$

So $3 \iint_R dA = 3\pi$ is the circulation of \mathbf{F} .

Problem 2. Compute the divergence of $\mathbf{F} = \langle x, x + y^2, xz + e^z \rangle$

$$\begin{aligned} \operatorname{div} \mathbf{F} &= \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(x + y^2) + \frac{\partial}{\partial z}(xz + e^z) \\ &= 1 + 2y + x + e^z \end{aligned}$$