Quiz 9 MATH 210, CALCULUS III, SUMMER 2015

NAME:

Problem 1. Find the critical points of

$$f(x,y) = 4 + 2x^2 + 3y^2$$

Use the second derivative test to classify them.

$f_x = 4x$ $\quad 4x = 0$ $\quad x = 0$

$f_y = 6y$ $\quad 6y = 0$ $\quad y = 0$

critical point $(0,0)$

$f_{xx} = 4 \quad f_{yy} = 6 \quad f_{xy} = 0$

$D(x,y) = 4 \cdot 6 - 0^2 = 24$

$D(0,0) = 24 > 0$

$f_{xx}(0,0) = 4 > 0$

so $\boxed{(0,0) \text{ is a local min}}$

Problem 2. Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = x + 2y$ subject to the constraint $x^2 + y^2 = 4$. $g(x,y) = x^2 + y^2 - 4$

$\nabla f(x,y) = \langle 1, 2 \rangle$

$\nabla g(x,y) = \langle 2x, 2y \rangle$

Equations:

$1 = \lambda \cdot 2x \longrightarrow x = \frac{1}{2\lambda}$

$2 = \lambda \cdot 2y \longrightarrow y = \frac{1}{\lambda}$

$x^2 + y^2 - 4 = 0$

$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 - 4 = 0$

$\frac{1}{4\lambda^2} + \frac{1}{\lambda^2} - 4 = 0$

$1 + 4 - 16\lambda^2 = 0$

$\lambda^2 = \frac{5}{16}$

$\lambda = \pm \frac{\sqrt{5}}{4}$

Check: $\left(\frac{1}{2 \cdot \frac{\sqrt{5}}{4}}, \frac{1}{\frac{\sqrt{5}}{4}}\right) = \left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

and $\left(\frac{1}{2 \cdot \frac{-\sqrt{5}}{4}}, \frac{1}{\frac{-\sqrt{5}}{4}}\right) = \left(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$

$f\left(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}} + 2 \cdot \frac{4}{\sqrt{5}} = \frac{10}{\sqrt{5}}$ max value

$f\left(\frac{-2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right) = -\frac{2}{\sqrt{5}} - 2 \cdot \frac{4}{\sqrt{5}} = -\frac{10}{\sqrt{5}}$ min value.