

## PROJECT SUMMARY

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**Proposed activities.** The PI proposes to use  $\mathcal{D}$ -module theory to investigate the structure of local cohomology modules as well as invariants of algebraic objects defined using local cohomology. The  $\mathcal{D}$ -module viewpoint has been exceptionally fruitful in the quest to better understand local cohomology in the characteristic-zero setting. The PI proposes the following specific projects:

- Hartshorne constructed de Rham homology and cohomology theories for local rings containing a characteristic-zero field. The PI, in his thesis, gave purely local proofs (using  $\mathcal{D}$ -modules) for the well-definedness and finite-dimensionality of these objects, not only for the homology and cohomology spaces but for the associated “Hodge-de Rham” spectral sequences as well. An important open question is whether duality between homology and cohomology holds at the level of the spectral sequences; the PI conjectures this is true and proposes to find a purely local proof.
- *Lyubeznik numbers* are invariants of local rings containing a field, defined using local cohomology. In positive characteristic, their definition can be extended to produce invariants of projective varieties, as proved by Zhang. The PI proposes a strategy for extending this result to all characteristics (obtaining thereby a new set of invariants for all projective varieties) based on techniques of Hartshorne. The PI’s thesis adviser was Gennady Lyubeznik and his sponsoring scientist at UIC is Wenliang Zhang, equipping the PI well to pursue this research proposal.
- It is conjectured that whenever  $R$  is a regular ring, any local cohomology module  $H_i^j(R)$  must have finitely many associated primes. An important open case is when  $R$  contains a field of characteristic zero, where the finiteness is known only if  $R$  is local. Puthenpurakal has recently made progress on this case for non-local  $R$  and the PI proposes further work in this direction.

**Intellectual merit.** Previous work on Lyubeznik numbers of projective varieties has focused largely on the question of their existence. If the PI’s second proposal above is successful, this chapter will be closed, enabling researchers to focus attention instead on learning what these numbers reveal about the varieties giving rise to them. Although local cohomology modules have been well-studied as  $\mathcal{D}$ -modules, their concomitant de Rham complexes have not been widely used. The PI used the de Rham complexes of local cohomology to give a one-paragraph proof that the de Rham cohomology (in Hartshorne’s sense) of a complete local ring is finite-dimensional, and he expects that his first proposal above will lead to further understanding and appreciation of this viewpoint.

**Broader impacts.** The PI has worked extensively with the University of Minnesota Talented Youth Mathematics Program (UMTYMP) as a teaching assistant and workshop leader. The PI also has serious interest in and experience with courses designed as transitions to advanced mathematics, and has worked with the training program at Minnesota for new teaching assistants. The PI remains interested in working with secondary students as well as teaching (and ideally, designing) first courses in abstract mathematics, and intends to seek out opportunities at UIC to do both.