

1. Consider the following tournament bracket listing the games in the 2015 Womens' NCAA basketball Sweet 16.

| | | | |
|---------|----------------------|--------------------------------|------------------|
| | Connecticut Texas | Notre Dame Stanford | |
| ALBANY | | | OKLAHOMA CITY |
| | Louisville Dayton | Iowa Baylor | |
| | Maryland Duke | S. Carolina N. Carolina | |
| SPOKANE | | | GREENSBORO |
| | Gonzaga Tennessee | Arizona State Florida State | |

- a. Let A be the event that Louisville and Stanford play in the Albany - Oklahoma City regional final. List all the elements (in terms of game outcomes) in A .
- b. Let B be the event that Florida State wins the Greensboro region. List all elements in B .
- c. Are A and B mutually exclusive? Why or why not?
- d. The Elite 8 consists of the eight teams that win in the first round after the Sweet 16. The Final Four consists of the four teams remaining two rounds after the Sweet 16. Count how many possible Elite 8s there are. Count how many possible Final Fours there are.

Solutions:

- a. One element is $\{Louisville\ beats\ Dayton,\ Connecticut\ beats\ Texas,\ Louisville\ beats\ Connecticut,\ Stanford\ beats\ Notre\ Dame,\ Iowa\ beats\ Baylor,\ Stanford\ beats\ Iowa\}$. Abbreviate this as $\{LD, CT, LC, SN, IB, SI\}$. Then, the other elements are $\{LD, TC, LT, SN, IB, SI\}$, $\{LD, CT, LC, SN, BI, SB\}$, and $\{LD, TC, LT, SN, BI, SB\}$.
- b. Similar to part a, we have $B = \{\{FA, NS, FN\}, \{FA, SN, FS\}\}$.
- c. Mutually exclusive means that if A happens, then B cannot happen, and vice versa. Actually, A and B are not mutually exclusive events, but they are *independent* events because the games occur in different parts of the bracket and do not influence each other.
- d. There are 8 independent games in the Sweet 16 round, each with 2 possible outcomes. So, there are 2^8 possible combinations of teams that can make up the Elite 8. Similarly, in order to pick the Final Four, we must select one team from each region. There are four regions each with four teams, so there are 4^4 possible combinations of teams that can make up the Final Four.

2. Consider the literary works: Hamlet, War and Peace, The Grapes of Wrath, Death of a Salesman, and The Odyssey. You randomly select a work to read for fun. a. List all elements of the event $A = a \text{ play is selected}$.
 b. List all elements of the event $B = a \text{ work of Shakespeare is selected}$.
 c. What is the relationship between events A and B ? d. Consider generic events X and Y . Use Venn diagrams to show that $(X \cup Y)' = X' \cap Y'$ and $(X \cap Y)' = X' \cup Y'$. These are called *DeMorgan's Laws*.

Solutions:

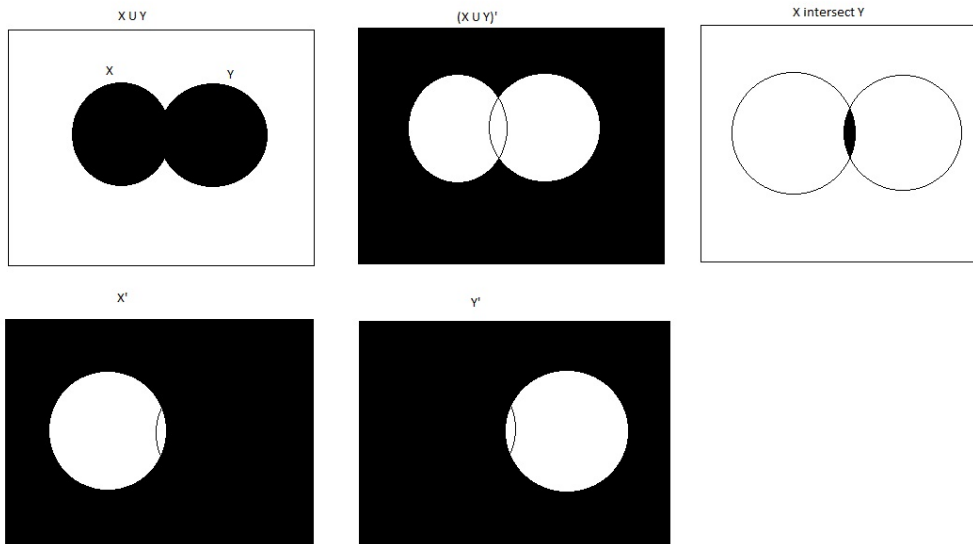
- a. You guys really need to brush up on your classical literature...

$A = \{Hamlet, Death of a Salesman\}$.

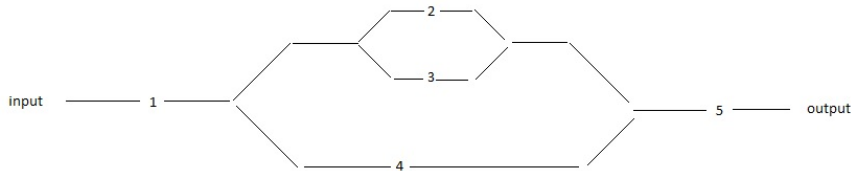
b. $B = \{Hamlet\}$.

c. B is a (proper) subset of A , $B \subset A$.

d. Use the pictures below:



3. Consider a manufacturing system with some components in series and some in parallel. The system works if there is at least one unbroken path from input to output.



- Find all elements of the event $A = \{ \text{at least 3 out of 5 components work} \}$.
- Find all elements of the event $B = \{ \text{at least 3 components work} \}$.
- Find all elements of the event $C = \{ \text{system functions} \}$.
- Find $B \cap C$.
- Find an event mutually exclusive to C .

Solutions:

- $A = \{ \{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 3, 4\}, \{1, 3, 5\}, \{1, 4, 5\}, \{2, 3, 4\}, \{2, 3, 5\}, \{2, 4, 5\}, \{3, 4, 5\} \}$.
- $B = A \cup \{ \{1, 2, 3, 4\}, \{1, 3, 4, 5\}, \{1, 2, 4, 5\}, \{1, 2, 3, 5\}, \{2, 3, 4, 5\}, \{1, 2, 3, 4, 5\} \}$.
- $C = \{ \{1, 2, 5\}, \{1, 3, 5\}, \{1, 4, 5\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{1, 2, 3, 4, 5\} \}$.
- $B \cap C = C$.
- There are many such events. One example is $\{2, 3, 5\}$.

4. You have 5 6-sided fair die.
- You roll all 5 die and record the sum. What are the possible outcomes?
 - You roll all five and record the value of each, paying no attention to order. Find all elements in the event $B = \{ \text{you roll exactly three 3's and the sum is greater than 15} \}$.
 - What are the elements of the event $C = \{ \text{all die turn up odd} \}$.
 - Find $B \cap C$.

Solutions:

- $\{5, 6, 7, \dots, 30\}$.
- $\{\{3, 3, 3, 1, 6\}, \{3, 3, 3, 2, 6\}, \{3, 3, 3, 4, 6\}, \{3, 3, 3, 5, 6\}, \{3, 3, 3, 6, 6\}, \{3, 3, 3, 2, 5\}, \{3, 3, 3, 4, 5\}, \{3, 3, 3, 5, 5\}, \{3, 3, 3, 4, 4\}\}$.
- $\{\{1, 1, 1, 1, 1\}, \{1, 1, 1, 1, 3\}, \{1, 1, 1, 1, 5\}, \{1, 1, 1, 3, 3\}, \{1, 1, 1, 3, 5\}, \{1, 1, 1, 5, 5\}, \{1, 1, 3, 3, 3\}, \{1, 1, 3, 3, 5\}, \{1, 1, 3, 5, 5\}, \{1, 1, 5, 5, 5\}, \{1, 3, 3, 3, 3\}, \{1, 3, 3, 3, 5\}, \{1, 3, 3, 5, 5\}, \{1, 3, 5, 5, 5\}, \{1, 5, 5, 5, 5\}, \{3, 3, 3, 3, 3\}, \{3, 3, 3, 3, 5\}, \{3, 3, 3, 5, 5\}, \{3, 3, 5, 5, 5\}, \{3, 5, 5, 5, 5\}, \{5, 5, 5, 5, 5\}\}$
- $\{3, 3, 3, 5, 5\}$

5. You have a standard 52-card playing deck.
- Let A be the event that you draw a random card and it turns out to be a club. List all elements of A .
 - Let B be the event that you draw a random card and it turns out to be a two. List all elements of B .
 - Find $A \cup B$ and $A \cap B$. Are A, B mutually exclusive?
 - Let C be the event that you draw a spade. Are A and C complements?
 - You and a friend are playing a version of poker. You each hold two cards, and there are five cards in play. You both form your best five card hand using your two cards and three of the cards in play. Suppose that you have a flush in hearts and your friend has a full house. What are the possible suits of the nine cards in question?

Solutions:

- $A = \{ Ac, 2c, 3c, 4c, 5c, 6c, 7c, 8c, 9c, 10c, Jc, Qc, Kc \}$ where 'c' stands for 'clubs'.
- $B = \{ 2c, 2h, 2s, 2d \}$.
- $A \cup B = \{ Ac, 2c, 3c, 4c, 5c, 6c, 7c, 8c, 9c, 10c, Jc, Qc, Kc, 2h, 2s, 2d \}$.
 $A \cap B = \{ 2c \}$. A and B are not mutually exclusive; their intersection is nonempty.
- A and C are not complements. A' would consist of all spades, hearts, and diamonds.
- A flush in hearts occurs when you have a five card hand of all hearts. A full house occurs when you have 3 of a kind (like 3 twos) and 2 of another kind (like a pair of aces). Therefore, a hand forming a full house has at least 3 suits represented. Since a full house beats a flush, your friend may be able to form a flush with his cards and the play cards, but he chooses not to. One possibility is: you have {heart, any suit}, in play {four hearts, any non-heart suit}, friend has {no hearts}. Another possibility is: you have {two hearts}, in play {three hearts, two of the same non-heart suit}, friend has {one each of the suits not represented in the 5 play cards}. The last possibilities are: you have {two hearts}, in play {three hearts, one each of two different non-heart suits}, friend has one of the following: {any two different non-heart suits}, {two of the same non heart suit not represented in the play cards}, or {one heart and any non-heart suit}.