1. Exercise 1.14 in the text. When you calculate sample variance, do so by hand and show the values of $(x_i - \bar{x})$, $\sum_{i=1}^n x_i^2$ and $\sum_{i=1}^n x_i$. Use both formulas for sample variance, i.e. $\frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x})^2$ and $\frac{n}{n-1}[\frac{1}{n}\sum_{i=1}^n x_i^2 - (\frac{1}{n}\sum_{i=1}^n x_i)^2]$ and show they give the same result.

   Solution:

   The values of $(x_i - \bar{x})$ are 1.5, 1.5, 2.5, -2.5, -1.5, 4.5, -5.5, -0.5. The values of the sum and sum of squares are $\sum_{i=1}^n x_i = 4564$ and $\sum_{i=1}^n x_i^2 = 2603832$. Using the first sample variance formula, we calculate

$$\frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x})^2 = \frac{1}{7}(1.5^2 + 1.5^2 + 2.5^2 + (-2.5)^2 + (-1.5)^2 + 4.5^2 + (-5.5)^2 + (-0.5)^2) = 10.$$

   Using the second formula, we calculate

$$\frac{n}{n-1}[\frac{1}{n}\sum_{i=1}^n x_i^2 - (\frac{1}{n}\sum_{i=1}^n x_i)^2] = \frac{8}{7}[\frac{2603832}{8} - (\frac{4564}{8})^2] = 10.$$

2. Exercise 1.16 in the text. Also, explain what insight you have gained regarding the formula for sample variance (hint: why do we divide by $n-1$ instead of n?).

   Solution:

$$\sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n (x_i) - n\bar{x}$$
$$= n\bar{x} - n\bar{x} = 0$$

   because $\bar{x} = \frac{1}{n}\sum_{i=1}^n x_i$.

   Here is the intuition: Since the above sum is always zero, it means that if you are given values for $x_1 - \bar{x}, x_2 - \bar{x}, ..., x_{n-1} - \bar{x}$, then the last value $x_n - \bar{x}$ is determined, i.e. $x_n - \bar{x} = -\sum_{i=1}^{n-1}(x_i - \bar{x})$. So, the last "piece of information" doesn't really contain any new information. That is one way of understanding why we divide by $n-1$; we only average over the number of pieces of unique information.

3. Exercise 2.10 in the text. The answer to part c) is supposed to be a sentence explaining in words what the given event means.

   Solution:

   a) The elements are $\{FFF\}$, $\{FFN\}$, $\{FNF\}$,$\{NFF\}$,$\{FNN\}$, $\{NFN\}$, $\{NNF\}$, $\{NNN\}$.

   b) $\{FFF\}$, $\{FFN\}$, $\{FNF\}$, $\{NFF\}$

   c) Probably the best answer is "the event that the second river is safe for fishing".

4. Exercise 2.26 in the text.

   Solution:

   a) $\binom{7}{5} = \frac{7!}{5!2!}$.

   b) $\binom{5}{3} = \frac{5!}{3!2!}$.

5. Exercise 2.29 in the text.

   Solution:

   $3 * 5 * 7 * 2$ test runs are needed.

6. Exercise 2.37 in the text.

   Solution:

   The boys and girls must sit like

$$GBGBGBGBG.$$

   From left to right, for the first position there are 5 girls to choose from. For the second position there are 4 boys to choose from. For the third position, 4 girls, then 3 boys, and so on... The number of ways to seat everyone is

$$5 * 4 * 4 * 3 * 3 * 2 * 2 * 1 * 1.$$

7. Exercise 2.45 in the text.

   Solution:

   Think of a bag with 3 I's, 2 N's, 1 F, 1 T, and 1 Y. Then, how many distinct orderings of those letters can you make if you sample them without replacement from the bag and use all 8 letters? We did an example like this in class. The answer is

$$\frac{8!}{3!2!}.$$

   If you had 8 distinct objects, then the answer would be 8!. Since there are 3 repeats of I and 2 repeats of N, you have to discard orderings of the letters that have I's and N's in the same positions, since they are indistinguishable.