1. Exercise 2.74 in the text.

   Solution:

   We want to calculate the conditional probability P(this randomly chosen student is a senior | this randomly chosen student has an A). Recall that the symbol | in the probability statement means "given" or "conditional on". The conditional probability is the ratio of the probability of the intersection to the marginal probability that the student has an A. The probability of the intersection is the probability that a senior has an A, which is 10/50 and the marginal probability of getting an A is 18/50, so that the conditional probability is 10/18.

2. Exercise 2.76 in the text.

   Solution:

   a) The probability of being a heavy smoker is 49/180. The probability of being both a heavy smoker and experiencing hypertension is 30/180. Therefore, the conditional probability is 30/49.

   b) Similarly, the conditional probability is $48/(48 + 26 + 19)$.

3. Exercise 2.80 in the text.

   Solution:

   The wording in this question is a little funny. The funny part is the sentence "The probability that an automobile being filled with gas also needs an oil change is 0.25." If we take this to mean that there are three different events: needing gas, needing oil, and needing a filter, then the first probability sounds like $P(g \text{ and } o)$. But, if we treat it as such, we don't have enough information to finish the problem...

Instead, let's take it to mean that the probability the car needs oil is $P(o) = 0.25$, the probability the car needs a filter is $P(f) = 0.4$ and the probability it needs both is 0.14. The part about gas is irrelevant (the car is getting gas for sure in any case). The we have for parts a) and b), $P(f|o) = 0.14/0.25$ and $P(o|f) = 0.14/0.40$.

4. Exercise 2.90 in the text.

   Solution:

   First, we calculate $P(B) = P(B|A)P(A) + P(B|A')P(A') = 0.75 * 0.3 + 0.2 * 0.7 = 0.365$. Also, $P(A \cap B) = P(B|A)P(A) = 0.75 * 0.3 = 0.225$. Next, $P(B \cap A') = P(B|A')P(A')0.2 * 0.7 = 0.14$. Now, for a) we can calculate $P(A \cap B \cap C) = P(C|A \cap B)P(A \cap B) = 0.2 * 0.225 = 0.045$. For part b) calculate $P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.225 = 0.075$. Also, $P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) = 1 - 0.3 - 0.365 + 0.225 = 0.56$. Finally, $P(B' \cap C) = P(B' \cap A \cap C) + P(B' \cap A' \cap C) = 0.8 * 0.75 + 0.9 * 0.56 = .564$. For part c), we again use the law of total probability and calculate $P(C) = P(C|A \cap B)P(A \cap B) + P(C|A' \cap B)P(A' \cap B) + P(C|A \cap B')P(A \cap B') + P(C|A' \cap B')P(A' \cap B') = 0.2 * 0.25 + 0.15 * 0.14 + 0.8 * 0.075 + 0.9 * 0.56 = .635$. The last part asks us to calculate $P(A|C \cap B')$ which is $\frac{0.8 * 0.075}{0.564}$.

5. Exercise 2.96 in the text.

   Solution:

   The times the traps are operated are independent from the chance the person is passing by the trap, hence we multiply probabilities and find that

$$P(ticket) = 0.4 * 0.2 + 0.3 * 0.1 + 0.2 * 0.5 + 0.3 * 0.2.$$

6. Exercise 2.102 in the text. Note: This is a very famous problem that has fooled people for years. In fact, this problem has been named after a game show host from long ago...

   The Monty Hall Problem:

   Here's the trick...

   Suppose you pick door A. Monty can always show you a losing door, B or C, since only one door has the prize. So, this doesn't tell you anything about A. In fact, $P($ A has the prize | Monty shows me a losing door$)$ = 1/3 because of the following table:

A	B	C
win	lose	lose
lose	win	lose
lose	lose	win

   What this means is that when Monty Hall reveals a door to you, the information he gives you is not about the door you picked, it's about the two doors you didn't pick. Therefore, $P($ switching wins$) = 2/3$. See that in the first row, if you switch, you lose, but you would win in the second or third rows! So, you should switch doors.

7. Exercise 2.103 in the text.

   Solution:

   Let G be Guilty, NG be Not Guilty, JG be Judged Guilty, and JNG be

Judged Not Guilty. Then, we are given $P(G) = 0.05$, $P(NG) = 0.95$, $P(JG|NG) = 0.01$, $P(JG|G) = 0.90$, and $P(JNG|G) = 0.1$. We are asked to find $P(NG|JG)$. Use Bayes Theorem and the law of total probability to get

$$P(NG|JG) = \frac{P(JG|NG)P(NG)}{P(JG)}$$

$$= \frac{P(JG|NG)P(NG)}{P(JG|G)P(G) + P(JG|NG)P(NG)} = \frac{0.01 * 0.95}{0.9 * 0.05 + 0.01 * 0.95} = 0.1743...$$