1. 3.4

   The sample points asked for are: HHH, $THHH$, $HTHHH$, $TTHHH$, $TTTHHH$, $HTTHHH$, $HHTHHH$, $THTHHH$. This is a discrete sample space because each sample point has a countable number of elements.

2. 3.8

   The random variable is $W = \#H - \#T$ and we know that $P(H) = 2/3$, $P(T) = 1/3$. Then, calculate

$$P(W = -3) = P(TTT) = (1/3)^3$$

$$P(W = -1) = P(HTT, THT, TTH) = 3(1/3)^2(2/3)$$

$$P(W = 1) = P(HHT, HTH, THH) = 3(2/3)^2(1/3)$$

$$P(W = 3) = P(HHH) = (2/3)^3.$$

3. 3.34

   (a) I think the easiest way to "show" this is to enumerate the probabilities for $Y = 0, ..., 5$. Think of the five samples as Bernoulli events with a 1 meaning "meets specification" and a 0 meaning "does not meet specification." Then, a string 01010 is interpreted as "the second and fourth samples meet specification."

$$P(Y = 0) = P(00000) = 0.01^5$$

$$P(Y=1)=P(10000,01000,00100,00010,00001)=5(0.01)^4(0.99)$$

$$P(Y=2)=P(11000,10100,10010,10001,01100,01010,01001,00110,00101,00011)$$

$$=10(0.01)^3(0.99)^2$$

$$P(Y=3)=P(00111,01011,01101,01110,10011,10101,10110,11001,11010,11100)$$

$$=10(0.01)^2(0.99)^3$$

$$P(Y=4)=P(01111,10111,11011,11101,11110)=5(0.01)(0.99)^4$$

$$P(Y=5)=P(11111)=0.99^5.$$

You will recognize these as equal to the binomial probabilities in the question.

(b) From above, $P(3 \text{ outside specifications}) = 9.8 \times 10^{0-6}$. If three out of a random five are outside specifications, it seems likely that the probability of meeting specifications is much less than 0.99.

4. 4.5

   Recall that for a standard 52 card deck, there are 4 Jacks, 4 Queens, 4 Kings, and 4 Aces. The expected winnings are $3 * \frac{8}{52} + 5 * \frac{8}{52} \approx \$1.23$. So, you should pay this much to play the game. If you pay more, you will expect to lose money when you play. If you pay less, well, no one would offer you to play for less money. You see, in either case, the game would be unfair.

5. 4.36

The expectation is $E(X) = 1*.3+2*.2+3*.1 = 1$ and $E(X^2) = 1*.3+4*.2+9*.1 = 2$ whence the variance can be calculated $Var(X) = E(X^2) - E(X)^2 = 2 - 1^2 = 1$.

6. Guided `R` exercise.

   First, use the partial sum formula for Geometric series to calculate a.

$$\sum_{k=0}^{20} a(\frac{1}{2})^k = a\frac{1-(\frac{1}{2})^21}{1-\frac{1}{2}} = 1$$

   where the last equality follows from the fact that the sum is a sum of probabilities so must sum to 1. Hence, $a \approx 1/2$. Now, here's the `R` code I used for the mean and variance.

```
> expected.value = function(){
+ a = 1/2
+ s = 0
+ for(k in 0:20){
+ s = s + k*a*((1/2)^k)
+ }
+ return(s)
+ }
> expected.value()
[1] 0.9999895
>
> expected.value2 = function(){
+ a = 1/2
```

```
+ s = 0
+ for(k in 0:20){
+ s = s + (k^2)*a*((1/2)^k)
+ }
+ return(s)
+ }
> expected.value2()
[1] 2.999768
> 2.99968-(.9999895^2)
[1] 1.999701
```

The population mean and variance are about 1 and 2. Let's look at the sample means and variances (yours could vary quite a bit from mine):

```
> p_k = function(k){
+ a = 1/2
+ value = a*((1/2)^k)
+ return(value)
+ }
> my.sample = function(n){
+ p = p_k(0:20)
+ s = sample.int(21, size = n, replace = TRUE, prob = p)-1
+ return(s)
+ }
> s1 = my.sample(10)
```

```
> s2 = my.sample(50)
> s3 = my.sample(100)
> s4 = my.sample(500)
> s5 = my.sample(1000)
> mean(s1)
[1] 1
> var(s1)
[1] 1.555556
> mean(s2)
[1] 1.1
> var(s2)
[1] 2.132653
> mean(s3)
[1] 0.76
> var(s3)
[1] 1.032727
> mean(s4)
[1] 1.066
> var(s4)
[1] 2.246136
> mean(s5)
[1] 0.968
> var(s5)
[1] 1.816793
```

In general, the sample means stay close to 1 ranging from 0.76 to 1.1, but the sample variances seem to *vary* more than the sample means, ranging from 1.03 to 2.24. The sample values should approach 1 and 2 as n gets large.