1. Exercise 3.40 in the text.

   Solution:

   (a) Integrate over Y to get the marginal density of X.

   $$\int_0^1 \frac{2}{3}(x+2y)dy = \frac{2}{3}(xy+y^2)|_0^1 = \frac{2}{3}(x+1) = \frac{2}{3}(x+1).$$

   (b) Integrate over X to get the marginal density of Y.

   $$\int_0^1 \frac{2}{3}(x+2y)dx = \frac{2}{3}(x^2/2+2xy)|_0^1 = \frac{2}{3}(\frac{1}{2}+2y).$$

   (c) $P(X \leq 1/2) = \int_0^{1/2} \frac{2}{3}(x+1)dx = \frac{2}{3}(x^2/2+x)|_0^{1/2} = \frac{2}{3}(\frac{1}{8}+\frac{1}{2}) = .41667.$

2. Exercise 3.42 in the text.

   Notice that the joint density is a product $e^{-(x+y)} = e^{-x}e^{-y}$ so that X and Y are independent. Hence, we can use the marginal distribution of X to calculate the conditional probability

   $$P(0 < X < 1|Y = 2) = \int_0^1 e^{-x}dx = -e^{-x}|_0^1 = 1 - e^{-1}.$$

3. Exercise 3.50 in the text. Additionally, do part (c) Find $P(Y = 3|X = 4)$

   (a) $P(X = 2) = 0.4$ and $P(X = 4) = 0.6$.

   (b) $P(Y = 1) = 0.25$, $P(Y = 3) = 0.5$, and $P(Y = 5) = 0.25$.

   (c) $P(Y = 3|X = 4) = 0.3/0.6 = 0.5$.

4. Exercise 4.44 in the text. Also, find the correlation. The solutions in the back of the book give the joint distribution referred to in problem

3.39. From this, we can calculate $E(X) = \frac{30}{70} + \frac{60}{70} + \frac{15}{70} = 1.5$, $E(Y) = \frac{40}{70} + 2 * \frac{15}{70} = 1$, and $E(XY) = \frac{18}{70} + \frac{54}{70} + \frac{6}{70} + \frac{12}{70} = 9/7$. This gives $Cov(X,Y) = E(XY) - E(X)E(Y) = -.214$. We can also calculate $E(X^2) = \frac{30}{70} + 4 * \frac{30}{70} + 9 * \frac{5}{70} = 2.785714$. And, $E(Y^2) = \frac{40}{70} + 4 * \frac{15}{70} = 10/7$. Therefore, $\sigma_Y \approx 0.655$ and $\sigma_X \approx 0.732$. The correlation between X and Y is $\frac{-0.214}{0.655*0.732} \approx -0.45$.

5. Exercise 4.46 in the text. Also, find the correlation. In exercise 3.44 we must find k such that

$$\int_{30}^{50} \int_{30}^{50} k(x^2 + y^2) dx dy = 1$$

You should see that $k = 1/1306667$. Then, we need to calculate $E(X)$, $E(Y)$, $E(X^2)$, $E(Y^2)$, and $E(XY)$. However, we really only need calculate $E(X)$, $E(X^2)$, and $E(XY)$ because $E(X) = E(Y)$ and $E(X^2) = E(Y^2)$ due to the symmetry of $f(x,y)$.

$$E(XY) = \int_{30}^{50} \int_{30}^{50} kxy(x^2 + y^2) dx dy$$

$$= k \int_{30}^{50} x^4 y/4 + x^2 y^3/2 |_{30}^{50} dy$$

$$= k[1360000 y^2/2 + 800 y^4/4 |_{30}^{50}] \approx 1665.$$

The other calculations are similar, giving $E(X) = E(Y) = 40.82$, $E(X^2) = E(Y^2) = 1699$. Then, we find $Var(X) = Var(Y) = 33.21$ and $Cov(X,Y) = -0.62$, so that $Corr(X,Y) = -0.62/33.21 = -0.02$, which is pretty low!

6. Exercise 4.70 in the text. (a) The joint density of X and Y does not factor into a function of X and a function of Y, so they are not independent. See that $f_Y(y) = 1/2 + 3/2y^2$ by integrating over X. Then, the conditional density is the ratio $f(X|Y) = \frac{3/2(x^2+y^2)}{1/2+3/2y^2}$ is a function of both x and y, so they are not independent.

   (b)

$$E(X+Y) = \int_0^1 \int_0^1 \frac{3}{2}(x+y)(x^2+y^2)dxdy = 3/2 \int_0^1 \int_0^1 x^3+x^2y+xy^2+y^3dxdy$$

$$= 3/2 \int_0^1 x^4/4 + x^3/3 * y + x^2/2 * y^2 + xy^3|_0^1 dy$$

$$= 3/2 \int_0^1 1/4 + y/3 + y^2/2 + y^3 dy$$

$$= 3/2[y/4 + y^2/6 + y^3/6 + y^4/4|_0^1]$$

$$= 3/2[1/4 + 1/6 + 1/6 + 1/4] = 3/2 * 5/6 = 5/4.$$

   Similarly, find $E(XY) = 3/8$.

   (c)

   By symmetry, $E(X) = E(Y)$, and $E(X^2) = E(Y^2)$. See that $E(X) = E(Y) = 5/8$. And, $E(X^2) = E(Y^2) = 7/15$. Then, $V(X) = V(Y) = \frac{7}{15} - (5/8)^2 = .07604167$. Then, $Cov(X,Y) = 3/8 - (5/8)^2 = -0.015625$ and $Corr(X,Y) = \frac{3/8-(5/8)^2}{0.07604167} = -0.205$.

   (d)

$E((X+Y)^2) = 3/2\int_0^1\int_0^1 (x+y)^2(x^2+y^2)dxdy$

$$= \frac{3}{2}(1.12222)$$

So that $Var(X+Y) = \frac{3}{2}(1.12222) - (5/4)^2 = 0.12083$.